MATH 122A EXAM REVIEW

Date: 1/30/22

Name: Hogarth

I want to make something very clear, these questions are my bias opinion of what questions I would study. The emphasis is on the I.
You should be able to site definitions and theorems by heart.

1. Find all the roots of $z^3 = \bar{z}$.

2. Find the roots of the following equation: $z^3 = (-1+i)$

3. Express $\left(\frac{-\sqrt{3}}{2}+\frac{i}{2}\right)^{603}$ in the form of $a+bi$.

4. what is the value of

$$(-2)^{\sqrt{2}}$$

on the principle branch.

5. What is a conformal map and what does it preserve?

6. What is so special about fractional linear transformations?

7. Find a fractional transformation that maps the points

$$z_1 = -1, \qquad z_2 = 0, \qquad z_3 = 1$$

onto the points

$$w_1 = -i, \qquad w_2 = 1, \qquad w_3 = i.$$

8. Show that the mapping

$$T(z) = \frac{(1-i)z+2}{(1+i)z+2}$$

maps the disk $D : |z+1| < 1$ onto the upper half plane $Im(T(z)) > 0$.

9. What does it mean for a function to be harmonic?

10. What is a harmonic conjugate of a function?

11. Show that $u(x,y) = xy^3 - x^3y$ is a harmonic function and find its harmonic conjugate $v(x,y)$.

12. What does it mean for a function to be analytic?

13. Define a region where $\sqrt{1-z}$ is analytic.

14. A holomorphic function f defined on a connected open set G must be constant on G if:

   (a) $f'(z) = 0$ for all $z \in G$
   (b) f takes only real values on G.
   (c) f lies on a line.
   (d) $\overline{f(z)}$ is a holomorphic function.
   (e) $|f|$ is constant on G.
   (f) at every point in G $f = 0$ or $f' = 0$. [Hint: consider f^2]

15. Suppose that f is analytic on the unit disc and that $Re f(z) = 3$ for all z in the unit disk. Then f is constant on the unit disk.

16. Let $z = x + iy$. The function defined by

$$f(z) = \frac{(\overline{z})^2}{z} = \frac{x^3 - 3xy^2}{x^2 + y^2} + i\frac{y^3 - 3x^2y}{x^2 + y^2}$$

when $z \neq 0$ and $f(0) = 0$ is *NOT* differentiable at the point $z_0 = 0$. However, the Cauchy-Riemann equations hold true at $(0,0)$. Why doesn't this contradict our theorem? Verify your claim.

17. Suppose u, v, U, V are harmonic functions, such that, v is a harmonic conjugate of u, and V is a harmonic conjugate of U. Show that $uV + vU$ is harmonic, and find its harmonic conjugate.

18. Suppose $f : \mathbb{C} \to \mathbb{C}$ is a non constant polynomial. If the derivative of f is never zero then f is injective. Is this true for arbitrary entire functions?