Final Review

Algebra

- Multiplying expressions out and simplifying (like section 1.2 problems). Also know how to do this when you first make a substitution (like problem 1.5.34)
- Factoring (using the quadratic formula)
- Working with fractions: getting a common denominator, adding, secret parentheses, multiplying (1.3)
- Solving systems of equations
  - Use one of the equations to eliminate one of the variables in another equation)
  - Check your answer works when you're done!
  - Know how to do this when there are other constants involved like a and b. See problem 4 on page 256.
- Find the inverse of a function $f(x)$. Example: $f(x) = 4x + 8$.
  1. Rewrite as $y = 4x + 8$
  2. Switch x and y: $x = 4y + 8$
  3. Solve for y: $y = \frac{x-8}{4}$.
  4. Rewrite as $f^{-1}(x) = \frac{x-8}{4}$.
  5. You can check your work by checking that $f(f^{-1}(x)) = x$:
     $f(f^{-1}(x)) = f(\frac{x-8}{4}) = 4\left(\frac{x-8}{4}\right) + 8 = x - 8 + 8 = x$.
- Also remember $f(a) = b \Leftrightarrow f^{-1}(b) = a$.
- Pythagorean Theorem problems (see 1.7.8 on page 31).
- Graph functions by plotting points or by using a table of values (like $y = x^2, y = x^3, y = 1/x$.

Ideas behind calculus

- Finding limits (see problem 5.1.2 on page 71)
- Summation notation (section 5.3)
- Find the slope of a line and equation of a line using slope-intercept form ($y = mx + b$) or point-slope form ($y - y_0 = m(x - x_0)$).
- Going from the parametric equations form of a line ($x =$ and $y =$ something in terms of t) to one of the two above forms.
- Finding where two lines intersect.
- Graphing lines.

Logarithms and Exponents

- log is the inverse of 10^x. $log(a) = b$ means $10^b = a$.
- ln is the inverse of e^x. $ln(a) = b$ means $e^b = a$.
- Log (and ln) rules
  1. $log(a \times b) = log(a) + log(b)$
  2. $log(a^p) = p \times log(a)$
  3. $log(a/b) = log(a) - log(b)$
  4. $log(1) = 0$
  5. $log(10^x) = x, ln(e^x) = x$
  6. $10^{log(x)} = x, e^{ln(x)} = x$
- Exponent rules
  1. $a^x \times a^y = a^{x+y}$
  2. $a^{-x} = 1/a^x$
  3. $(a^x)^y = a^{xy}$
  4. $a^0 = 1$
- Solving algebra problems using logs.
- Doing computations using log tables and graphs
- Doubling time and half life: amount after t years $= A \times 2^{t/K}$ or $A \times (1/2)^{t/K}$, where A is the initial amount and K is the doubling time/half life

Derivatives

- Slope of secant line = average rate of change in $f(t)$ between t_0 and $t_1 = \frac{f(t_1)-f(t_0)}{t_1-t_0}$
- Slope of tangent line = instantaneous rate of change of $f(t)$ when $t = t_0$ is
  $f'(t_0) = \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}$
- Derivative rules
  1. $(x^n)' = nx^{n-1}$
  2. $(C)' = 0$
  3. $(Cf(x))' = Cf'(x)$
  4. $(e^{Kx})' = Ke^{Kx}$
  5. $(ln(Kx))' = K/x$
- Finding the equation of the tangent line to $f(x)$ at $x = a$: Plug the slope $f'(a)$ and point $(a, f(a))$ into point-slope form, $y - y_0 = m(x - x_0)$.
- Calculating derivatives using limits

- $f'(x) > 0$ when $f(x)$ is increasing (slopes upwards).
- $f'(x) < 0$ when $f(x)$ is decreasing (slopes downwards).
- If $p(t) =$ position, then $p'(t) = v(t) =$ velocity and $p''(t) = v'(t) = a(t) =$ acceleration.
- If $f''(t) > 0$, the graph of $f(t)$ is concave up (looks like a cup opening upwards).
- If $f''(t) < 0$, the graph of $f(t)$ is concave down (looks like a cup opening downwards).

Finding (local) max/min of a $f(x)$

1. Figure out a constraint equation (eg, from info like "the area must be 5 ft^{2}") and use it to solve for one of the variables.

2. Write down the expression you want to minimize or maximize, which will have two variables.

3. Use step 1 to substitute into this expression and get everything in terms of one variable, say x. This gives us our $f(x)$.

4. Set $f'(x) = 0$ and solve for x. Say we get $x = a$.

5. If f' changes from negative to positive at a, there is a local min at $x = a$. If f' changes from positive to negative at a, there is a local max at $x = a$.

6. Plug a back into $f(x)$ to get the local min/max of $f(a)$.

Note that you usually don't need to worry about step 2, if you are told that $f(x)$ only has a max or only has a min.

For non word problems, you just have to do steps 3-6.

This is sometimes called the **first derivative test** for finding (local) max/min. There is also a **second derivative test**:

1. Set $f'(x) = 0$ and solve for x. Say we get $x = a$.

2. Compute $f''(x)$ and plug a in. If $f''(a) > 0, thereisalocalminat$a.$If f$f"(a)¡0, there is a local max at a.

3. Plug a back into $f(x)$ to get the local min/max of $f(a)$.