MATH 34A, Midterm 2 Practice Problems

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1. Solve $7^x = 2^{x+4}$ for x.

Solution: $log(7^x) = log(2^{x+4}) \Rightarrow xlog(7) = (x+4)log(2) \Rightarrow xlog(7) - xlog(2) = 4log(2) \Rightarrow x = \frac{4log(2)}{log(7)-log(2)}$

2. Combine the following into a single logarithm: $2ln(x) + 3ln(y) - ln(z)$.

Solution: $ln(x^2y^3/z)$

3. Split up the following into logarithms of a single variable: $log(xy/z^2)$.

Solution: $log(x) + log(y) - 2log(z)$

4. Simplify: (a) $e^{2ln(5)}$ (b) $log(10)$ (c) $10^{log(6)}$ (d) $log(10^{-3})$ (e) $10^{2+log(5)}$.

Solution: (a) 25 (b) 1 (c) 6 (d) -3 (e) 500

5. A population of rabbits is doubling every 2 years. In 1970 there were 500 rabbits. How many rabbits are there in 1980? When will there be 3000 rabbits?

Solution: $A \times 2^{t/K} = 500 \times 2^{t/2} = 500 \times 2^5 = 500 \times 32 = 16000$ rabbits in 1980.

$3000 = 500 \times 2^{t/2} \Rightarrow 6 = 2^{t/2} \Rightarrow log(6) = (t/2)log(2) \Rightarrow t = 2log(6)/log(2) = 5.17$ There will be 3000 rabbits in 1975.

6. Find the derivatives of: (a) $2/x^3$ (b) $5e^{4x}$ (c) $(x+1)(3x+2a)$ where a is a constant
(d) $(2x-c)^2$ where c is a constant (e) $x^{2e} + e^{2x}$

Solution: (a) $-6x^{-4}$ (b) $20e^{4x}$ (c) $6x + 3 + 2a$ (d) $8x - 4c$ (e)$2ex^{2e-1} + 2e^{2x}$

7. Suppose $f(t) = 2t^3 - 3t^2 - 12t + 1$ is the position in miles of a car after t hours.
(a) Find the velocity and acceleration of the car after 30 minutes.
(b) Where is $f(t)$ increasing?
(c) Where is $f(t)$ concave up?
(d) Find the average velocity between the 1st and 2nd hours.

Solution: (a) Velocity: $f'(t) = 6t^2 - 6t - 12 \Rightarrow f'(1/2) = (6/4) - 3 - 12 = -13.5$ mph.

Acceleration: $f''(t) = 12t - 6 \Rightarrow f''(1/2) = 0$ mph.

(b) $f'(t) = 6t^2 - 6t - 12 = 6(t^2 - t - 2) = 6(t-2)(t+1)$. $f(t)$ is increasing where $f'(t) > 0$: for $t < -1$ and $t > 2$.

(c) $f''(t) = 12t - 6 = 6(2t - 1)$. $f(t)$ is concave up where $f''(t) > 0$: for $t > 1/2$.

(d) $\frac{f(2)-f(1)}{2-1} = (16 - 12 - 24 + 1) - (2 - 3 - 12 + 1) = -19 + 12 = -7$ mph.

8. Find the equation of the tangent line to $f(x) = x^2$ at $x = 3$. Graph $f(x)$ and this tangent line.

Solution: $f'(x) = 2x$, so $f'(3) = 6$. A point on the tangent line is $(3, f(3)) = (3, 9)$ so we have $y - 9 = 6(x - 3) \Rightarrow y = 6x - 9$.

9. Let $f(x) = 2x^2 + 1$. Show that $f'(x) = 4x$ using the definition of derivative.

Solution: $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{2(x + \Delta x)^2 + 1 - (2x^2 + 1)}{\Delta x} =$

$$\lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 + 1 - 2x^2 - 1}{\Delta x} = \lim_{\Delta x \to 0} 4x + 2\Delta x = 4x + 2(0) = 4x$$