(yellow version. blue similar).

1 (A) for all $\varepsilon > 0$ there is a $\delta > 0$ so that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$

(B) We can make $f(x)$ get arbitrarily close to L by making x close enough (but not equal) to a.

(C) $\lim_{x \to a} f(x) = f(a)$

(D) ① $f(a)$ exists (a is in the domain of f)

② $\lim_{x \to a} f(a)$ exists

③ these two things are equal.

2 (A) $\ln\left(\dfrac{x^4 (x-1)^5}{(x^2+1)^2}\right)$

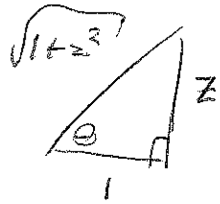
(B) $2^{\log_2 y + \log_4 (y+2)^2} = \cancel{2^{\log_2}} y \, 2^{\log_4 (y+2)^2}$

$= y\left((2^2)^{\log_4 y+2}\right)$

$= y\left(\cancel{4^{\log_4}}(y+2)\right) = y(y+2).$

(C) $\tan^{-1} z = \theta \iff \tan\theta = z$

$\tan = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{z}{1}$:



$(c^2 = a^2 + b^2)$

so $\sin(\tan^{-1} z) = \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{z}{\sqrt{1+z^2}}$

3 (A). f is not injective:

Write $f(x) = x(x-3)$ and note that $f(0) = f(3) = 0$

(B) $g(x) = 4e^{5x+1}$ $(x \in \mathbb{R})$ (g increasing $\Rightarrow$ injective)

$$y = 4e^{5x+1}$$

$$\frac{y}{4} = e^{5x+1}$$

$$\ln\left(\frac{y}{4}\right) = 5x+1$$

$$x = \frac{\ln\left(\frac{y}{4}\right) - 1}{5}$$

$$\boxed{g^{-1}(x) = \frac{\ln\left(\frac{x}{4}\right) - 1}{5}}$$

(C) $h(x) = F(cx)$

$$y = F(cx)$$

$F^{-1}(y) = \cancel{F^{-1}}(F(cx))$ (F^{-1} exists as F is injective)

$$F^{-1}(y) = cx$$

$$x = \frac{1}{c}F^{-1}(y) \quad (c \neq 0)$$

$$\boxed{h^{-1}(x) = \frac{1}{c}F^{-1}(x)}$$

4 (A) $\lim_{x \to 3} \frac{(x+2)\cancel{(x-3)}}{(x-1)\cancel{(x-3)}} = \frac{3+2}{3-1} = \boxed{\frac{5}{2}}$

(B) $\lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h \to 0} \frac{2-2-h}{2h(2+h)} = \lim_{h \to 0} \frac{-\cancel{h}}{2\cancel{h}(2+h)}$

$$= \frac{-1}{2(2+0)} = \boxed{\frac{-1}{4}}$$

4 (C) $\lim_{a\to 7^-} \frac{|a+7|}{a+7} = \lim_{a\to 7^-} \frac{-(\cancel{a+7})}{\cancel{a+7}}$ (since $a+7 < 0$ for $a < 7$)

$= \boxed{-1}$

(D) $\lim_{x\to\infty} \frac{\sqrt{8+3x^2}}{3+11x} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$ (since $\sqrt{x^2} = |x| = x$ for $x > 0$)

$= \lim_{x\to\infty} \frac{\sqrt{\frac{8}{x^2}+3}}{\frac{3}{x}+11} = \frac{\sqrt{0+3}}{0+11} = \boxed{\frac{\sqrt{3}}{11}}$

5. Let $f(x) = e^x + 2x - 3$. f is continuous on $[0,1]$.

Note that $f(0) = 1-3 = -2$ and $f(1) = e+2-3 = e-1 (>0)$.

Then since f is continuous on $[0,1]$ and 0 is between $f(0)$ and $f(1)$, by the IVT there is a number c in $(0,1)$ so that $f(c) = 0$.

$f(c) = 0$ means $e^c + 2c - 3 = 0$

$\longrightarrow e^c = 3 - 2c$, which is what we wanted to show.

6. We're given that (1) $\underbrace{-2t^2-4t+1}_{f(t)} \leq g(t) \leq \underbrace{3}_{h(t)}$ for all $t \in \mathbb{R}$.

Note that $\lim_{t\to -1} f(t) = \lim_{t\to -1} -2t^2 - 4t + 1 = -2(-1)^2 - 4(-1) + 1 = 3$

and $\lim_{t\to -1} h(t) = \lim_{t\to -1} 3 = 3$.

Then since (1) holds and $\lim_{t\to -1} f(t) = \lim_{t\to -1} h(t) = 3$,

$\lim_{t\to -1} g(t) = 3$ by the squeeze theorem.

7. (A) ① $f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\sqrt{3+h+1} - \sqrt{3+1}}{h}$

$= \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$

or

② $f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3}$

$= \lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3}$

(B) ① $f'(3) = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \lim_{h \to 0} \frac{\cancel{4}+\cancel{h}-\cancel{4}}{\cancel{h}(\sqrt{4+h}+2)}$

$= \frac{1}{\sqrt{4+0}+2} = \boxed{\frac{1}{4}}$

or

② $f'(3) = \lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x \to 3} \frac{x+1-4}{x-3(\sqrt{x+1}+2)}$

$= \lim_{x \to 3} \frac{\cancel{x-3}}{\cancel{x-3}(\sqrt{x+1}+2)}$

$= \frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$

(C) $f'(3) = \frac{1}{4}$ = slope of tangent line at $(3, f(3)) = (3, 2)$ so

$y - 2 = \frac{1}{4}(x-3)$

$\rightarrow \boxed{y = \frac{1}{4}x + \frac{5}{4}}$