Quiz 4

NAME:
PERM:
SECTION: T 8 AM / T 4 PM / T 5 PM / T 6 PM / TH 6 PM

1. Suppose that two functions f and g are both continuous at a point $x = a$ belonging to the intersection of their respective domains. Write a complete proof that the function fg given by $(fg)(x) = f(x) \cdot g(x)$ is also continuous at the point $x = a$. (Hint: Use the corresponding Limit Law.)

It helps to start by writing out the definitions:

f is continuous at $x=a$: $\lim_{x\to a} f(x) = f(a)$

g is continuous at $x=a$: $\lim_{x\to a} g(x) = g(a)$

We want to show $(fg)(x) = f(x)\cdot g(x)$ is continuous at $x=a$:

$$\lim_{x\to a} f(x)\cdot g(x) = f(a)\cdot g(a) \quad (= (fg)(a))$$

Proof: $\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$ (this limit law applies since $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist by definition of continuity at $x=a$)

$= f(a)\cdot g(a)$ (since f, g are continuous at $x=a$.)

Then since $\lim_{x\to a} f(x)\cdot g(x) = f(a)\cdot g(a)$, (fg) is continuous at $x=a$ by definition.

2. Using the limit definition of derivative, find $g'(3)$, where $g(x) = x^2 + 2x$.

1. $$g'(3) = \lim_{h\to 0} \frac{g(3+h) - g(3)}{h} = \lim_{h\to 0} \frac{(3+h)^2 + 2(3+h) - (3^2 + 2(3))}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{9} + 6h + h^2 + \cancel{6} + 2h - \cancel{15}}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 8h}{h} = \lim_{h\to 0} \frac{\cancel{h}(h+8)}{\cancel{h}} = 0 + 8 = \boxed{8}$$

or

2. $$g'(3) = \lim_{x\to 3} \frac{g(x) - g(3)}{x-3} = \lim_{x\to 3} \frac{x^2 + 2x - 15}{x-3}$$
$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+5)}{\cancel{x-3}} = 3 + 5 = \boxed{8}$$