

Quiz 6

Name:

Perm:

Section Day and Time:

1. Find $f'(x)$ if

$$f(x) = \sin(x)\sec(x)\cot(x) + \cos(x)\csc(x)\tan(x).$$

(Hint: Think before you differentiate)

$$f(x) = \cancel{\sin(x)} \left(\frac{1}{\cancel{\cos(x)}} \right) \left(\frac{\cancel{\cos(x)}}{\cancel{\sin(x)}} \right) + \left(\cancel{\cos(x)} \right) \left(\frac{1}{\cancel{\sin(x)}} \right) \left(\frac{\cancel{\sin(x)}}{\cancel{\cos(x)}} \right)$$

$$= 1 + 1 = 2$$

Then $f(x) = 2$ so $f'(x) = 0$

If you don't know your trig functions you need to use the product rule on both terms of $f(x)$, which will in turn require two more product rule applications each.

2. Find $f''(x)$ if

$$f(x) = \frac{x}{x^2-1}$$

There are many ways to do this. ① Writing $x^2-1 = (x+1)(x-1)$ avoids the chain rule.

② Write $f(x) = x(x^2-1)^{-1} \rightarrow$ product (+ chain) rule

③ Quotient rule: $f'(x) = \frac{(x^2-1)(1) - (x)(2x)}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2}$

so $f''(x) = \frac{(x^2-1)^2(-2x) - (-x^2-1)(2(x^2-1)(2x))}{(x^2-1)^4} = \frac{2x(x^2+3)}{(x^2-1)^3}$
 (after simplifying)

④ Logarithmic differentiation:

From $f'(x) = \frac{-x^2-1}{(x^2-1)^2}$ we have $\ln(f'(x)) = \ln\left(\frac{-x^2-1}{(x^2-1)^2}\right) = \ln(-x^2-1) - 2\ln(x^2-1)$

so $\frac{f'}{f} \cdot f''(x) = \frac{-2x}{-x^2-1} - \frac{4x}{x^2-1}$

$\rightarrow f''(x) = \left(\frac{-x^2-1}{(x^2-1)^2}\right) \left(\frac{-2x}{-x^2-1} - \frac{4x}{x^2-1}\right) = \text{simplify...}$