

Quiz 9

NAME:

PERM:

SECTION: T 8 AM / T 4 PM / T 5 PM / T 6 PM / TH 6 PM

1. Consider the function $f(x) = x^4 - 2x^2 + 1$ on the interval $[-1, 2]$.

(a) Find the absolute maximum value of $f(x)$ on this interval. At what x -value(s) does this absolute maximum occur?

(b) Find the absolute minimum value of $f(x)$ on this interval. At what x -value(s) does this absolute minimum occur?

$$f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 0 \quad (\text{never undefined})$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \text{Critical values: } & x=0 & x=-1, 1 \end{array}$$

$$f(0) = 1$$

$$f(-1) = (-1)^4 - 2(-1)^2 + 1 = 1 - 2 + 1 = 0$$

$$f(1) = 1 - 2 + 1 = 0$$

$$\text{Endpoints: } f(2) = 16 - 8 + 1 = 9 \quad \text{and} \quad f(-1) = 0.$$

So the absolute max is 9, when $x = 2$,
and the absolute min is 0, when $x = \pm 1$.

(c) What does the Mean Value Theorem tell us about $f(x)$ on this interval?

Since f is continuous on $[-1, 2]$ and differentiable on $(-1, 2)$, the MVT says that there is some number c in $(-1, 2)$ so that

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)} = \frac{9 - 0}{3} = 3.$$