MATH 3B: Final review problems

A. Volumes The region bounded by the given curves is rotated about the specificed axis. Find the volume of the resulting solid by any method.

(1) $y = x^4, y = 0, x = 1$; about $x = 2$
(2) $x = y^2 + 1, x = 2$; about $y = -2$
(3) $y = \sqrt{x-1}, y = 0, x = 5$; about the x-axis
(4) Find the volume of a sphere of radius r (using calculus).

B. Arc length

(1) Find the length of the arc of the curve $x = \frac{2}{3}(y-1)^{3/2}$ between $1 \leq y \leq 4$.
(2) Find the length of the arc of the curve $x = t - \sin t, y = 1 - \cos t$ for $0 \leq t \leq 2\pi$.

C. Surface area

(1) Find the surface area of the solid obtained by rotating the following parametric curve about the x-axis: $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq \frac{\pi}{2}$.
(2) Find the surface area of the solid obtained by rotating the curve $y = x^3, 0 \leq x \leq 2$ about the x-axis.
(3) Find the surface area of the solid obtained by rotating the curve $x = \frac{1}{3}(y^2+2)^{3/2}, 1 \leq y \leq 2$ about the x-axis. (hint: write the stuff under the radical as something squared).
(4) Find the surface area of a sphere of radius r (using calculus).

D. Area

(1) Find the area of an ellipse, $x = 3\cos t, y = 5\sin t, 0 \leq t \leq 2\pi$

E. Riemann sums Use Riemann sums to evaluate the following limits.

(1) $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k}{\sqrt{n^2+kn}}$
(2) $\lim_{n\to\infty} \left[\left(\frac{1}{n}\right)^1 \left(\frac{2}{n}\right)^2 \cdot \ldots \cdot \left(\frac{n}{n}\right)^n \right]^{1/n^2}$
(3) $\lim_{n\to\infty} \frac{1}{n\sqrt{n}} \sum_{k=1}^{n} \frac{k}{\sqrt{n+k}}$
(4) $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\frac{k}{n^3} + \frac{2}{n^2}}$
(5) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2} e^{k^2/n^2}$

F. Comparison test Use the comparison test to show that the following integrals converge or diverge.

(1) $\int_1^\infty \frac{x}{x^3+1}\,\mathrm{d}x$

(2) $\int_1^\infty e^{-x^2}\,\mathrm{d}x$

(3) $\int_1^\infty \frac{1+e^{-x}}{x}\,\mathrm{d}x$

(4) $\int_0^1 \frac{sec^2x}{x^{3/2}}\,\mathrm{d}x$

(5) $\int_0^1 \frac{sin^2x}{\sqrt{x}}\,\mathrm{d}x$

(6) $\int_{-\infty}^{-1} \frac{sin^2x}{x^2}\,\mathrm{d}x$

(7) $\int_1^\infty \frac{sin(x)+2}{x}\,\mathrm{d}x$

(8) $\int_0^1 \frac{sin(x)+2}{x^2}\,\mathrm{d}x$

(9) $\int_1^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,\mathrm{d}x$

G. Determine if the following improper integrals converge or diverge, and evaluate those that are convergent.

(1) $\int_0^1 \frac{1}{x}\,\mathrm{d}x$

(2) $\int_0^1 \frac{1}{x(lnx)^2}\,\mathrm{d}x$

(3) $\int_0^1 x^2 ln(x)\,\mathrm{d}x$

(4) $\int_0^1 \frac{ln(x)}{x}\,\mathrm{d}x$