Volume: We can find the volume of a solid by adding up areas of cross sections of the solid. The main formula is $\int_a^b A(x)\ \mathrm{d}x$ or $\int_c^d A(y)\ \mathrm{d}y$ where $A(x), A(y)$ give the area of a cross section of the solid. The two main cases are:
Disks/Washers: $A = \pi r^2$ or $\pi\big((r_{\text{outer}})^2 - (r_{\text{inner}})^2\big)$. Cross sections are perpendicular to the axis of rotation.
Cylindrical shells: $A = 2\pi rh$. Cross sections are parallel (she||s) to the axis of rotation.

Arc length: $L = \int ds$.

Surface area:
$S = 2\pi \int yds$ (rotation about x-axis)
$S = 2\pi \int xds$ (rotation about y-axis)

where ds, the arc length differential, is:

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx$ if $y = f(x), a \leq x \leq b$.
$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}dy$ if $x = g(y), c \leq y \leq d$.
$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt$ if $x = f(t), y = g(t), \alpha \leq t \leq \beta$.
$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}d\theta$ if $r = f(\theta), \alpha \leq \theta \leq \beta$.

Area of a region D (as below) in the xy-plane:
If $x = x(t), y = y(t), a \leq t \leq b$ then:
area of $D = \left|\frac{1}{2}\int_a^b x(t)y'(t) - y(t)x'(t)dt\right|$
where the boundary of D is traversed once as t increases from a to b.

Evaluating limits by interpreting them as Riemann sums
By definition, $\displaystyle\int_0^1 f(x)\ \mathrm{d}x = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$.
So whatever we're given, we want to write a summation with a $\frac{1}{n}$ out front and an expression with $\frac{k}{n}$'s inside.
The $\frac{k}{n}$'s turn into x's and we have our function to integrate from 0 to 1. Integrate to get the answer.

Improper integrals

Type 1: infinite interval: $\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$, $\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$

Type 2: discontinuity in interval: -

f discontinuous at a: $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$

f discontinuous at b: $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$

f discontinuous at $c, a < c < b$: $\int_a^b f(x)dx = \lim_{t\to c^-} \int_a^t f(x)dx + \lim_{t\to c^+} \int_t^b f(x)dx$

Comparison Test: If f, g are continuous with $f(x) \geq g(x) \geq 0$ for $x \geq a$, then:
(a) $\int_a^\infty f(x)dx$ convergent $\Rightarrow \int_a^\infty g(x)dx$ convergent (if a larger function f converges, so does g)
(b) $\int_a^\infty g(x)dx$ divergent $\Rightarrow \int_a^\infty f(x)dx$ divergent (if a smaller function g diverges, so does f)

There are analagous tests for integrals of the form $\int_{-\infty}^b f(x)dx$ and improper integrals with discontinuities.