MATH 3B Worksheet: Improper integrals

A. Determine if the following improper integrals converge or diverge, and evaluate those that are convergent.

(1) $\int_1^\infty \frac{1}{x} \, dx$

(2) $\int_1^\infty \frac{1}{x^2} \, dx$

(3) $\int_{-\infty}^\infty xe^{-x^2} \, dx$

(4) $\int_{-\infty}^\infty x \, dx$

(5) $\int_0^2 \frac{1}{x-1} \, dx$

(6) $\int_0^\infty x^3 e^{-x^2} \, dx$

(7) $\int_0^1 \frac{1}{x} \, dx$

(8) $\int_0^1 \frac{1}{x(lnx)^2} \, dx$

(9) $\int_0^1 x^2 ln(x) \, dx$

(10) $\int_0^1 \frac{ln(x)}{x} \, dx$

B. Identify which of the following integrals are improper.

(1) $\int_{-10}^{10} \frac{1}{x^3+1} \, dx$

(2) $\int_0^\pi \tan x \, dx$

(3) $\int_{-1}^1 \frac{1}{x^2-x-2} \, dx$

(4) $\int_0^2 \arctan x \, dx$

(5) $\int_1^{10} \ln|x-5| \, dx$

C. Use the comparison test to show that the following integrals converge or diverge.

(1) $\int_1^\infty \frac{x}{x^3+1} \, dx$

(2) $\int_1^\infty e^{-x^2} \, dx$

(3) $\int_1^\infty \frac{1+e^{-x}}{x} \, dx$

D. Find the values of p for which $\int_0^1 \frac{1}{x^p} \, dx$ converges and evaluate the integral for those values of p.