MATH 3B Worksheet: Riemann sums and definite integrals
Name:
Perm#:

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1. Consider the integral $\int_0^4 16 - t^2 \, dt$. Find the Riemann sum for this integral using:
   (a) right-hand sums for $n = 4$.
   (b) left-hand sums for $n = 4$.
   (c) Sketch a graph for (a)-(b). Which underestimates the actual value of the integral? Overestimates? What can you say in general?
   (d) Now suppose that $v(t) = 16 - t^2$ is the velocity (in ft/s) of a car after t seconds, so the car comes to a stop after 4 seconds. Suppose that a kitten is 45 feet in front of the car at $t = 0$. Based on (a)-(c), can we determine whether or not the car struck the kitten?
   (e) Compute the actual value of the integral to find that the kitten lives to see another day.

2. Evaluate the following integrals by interpreting them in terms of areas:

   (a) $\int_1^5 x + 1 \, \mathrm{d}x$.

   (b) $\int_{-2}^2 |2x + 2| \, \mathrm{d}x$.

   (c) $\int_2^6 -\sqrt{4 - (x-4)^2} \, \mathrm{d}x$.

3. Use the limit definition of the definite integral to compute the following:

   (a) $\int_0^2 2 \, \mathrm{d}x$.

   (b) $\int_0^2 x \, \mathrm{d}x$.

   (c) $\int_1^2 x + 1 \, \mathrm{d}x$.

   (d) $\int_0^2 x^2 + 1 \, \mathrm{d}x$.