

MATH 3B Worksheet: The Fundamental Theorem of Calculus

Name:

Perm#:

Suppose f is continuous on $[a, b]$.

FTC1: $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.

FTC2: If F is any antiderivative of f , then $\frac{d}{dx} \int_a^b f(x) dx = F(b) - F(a)$.

Since $F' = f$, we can rewrite this as $\frac{d}{dx} \int_a^b F'(x) dx = F(b) - F(a)$.

1. 3 examples of FTC1. Compute:

(a) $\frac{d}{dx} \int_0^x \sin(t^2) dt$.

(b) $\frac{d}{dx} \int_x^4 \sin(t^2) dt$.

(c) $\frac{d}{dx} \int_x^{x^3} \sin(t^2) dt$. (hint: Chain Rule!)

2. FTC2 tells us how to compute definite integrals. Examples:

(a) $\int_1^2 3x^2 + 2 \, dx$.

(b) $\int_{-1}^3 \frac{1}{x} \, dx$. (hint: trick question!)

3. Notice that FTC2 says in words that:

the integral of the rate of change of F represents the net change in F from $x = a$ to $x = b$.

This is called the **Net Change Theorem**. Some examples:

(a) If $v(t)$ gives the velocity of an object (in ft/s) after t seconds, what does $\int_0^5 v(t) \, dt$ represent? What does $\int_0^5 |v(t)| \, dt$ represent?

(b) If $V(t)$ gives the volume of a solid (in m^3) after t hours, what does $\int_0^{10} V'(t) \, dt + V(0)$ represent?