

MATH 3B Worksheet: u -substitution and integration by parts

Name:

Perm#:

u -substitution/change of variables - undoing the chain rule:

Given $\int_a^b f(g(x))g'(x) dx$, substitute $u = g(x) \Rightarrow du = g'(x) dx$ to convert

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du.$$

u -substitution works for integrating compositions of functions; pick u to be the 'inside' function (for indefinite integrals, drop the limits of integration).

1. Compute:

(a) $\int 2x \sin(x^2) dx.$

(b) $\int x^2 (3 - 5x^3)^4 dx.$

(c) $\int_{-\pi}^{\pi} \cos(x)\sin^{10}(x) dx.$

(d) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx.$

(e) $\int_2^e \frac{1}{x \ln(x)} dx.$

Integration by parts - undoing the product rule: $\int u dv = uv - \int v du$.

Generally, picking u in this descending order works, and dv is what's left:

Inverse trig

Logarithm

Algebraic (polynomial)

Trig

Exponential

2. Compute:

(a) $\int x e^x dx$.

(b) $\int x \sin(x) dx$.

(c) $\int_1^e \ln(x) dx$.

(d) $\int e^x \cos(x) dx$.

(e) $\int \sin(\sqrt{x}) dx$. (hint: first make a substitution)