

MATH 3B Worksheet: Cauchy-Schwarz inequality

Name:

Perm#:

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**Cauchy-Schwarz inequality:**  $\left(\int_a^b f(x)g(x)dx\right)^2 \leq \left(\int_a^b f(x)^2dx\right) \left(\int_a^b g(x)^2dx\right)$

1. Use the Cauchy-Schwarz inequality to show the following:

(a)  $\left(\int_0^1 (x+1)x dx\right)^2 \leq \frac{7}{9}$ .

(b)  $\left(\int_0^1 x^2 + 2x + 1 dx\right)^2 \leq \frac{49}{9}$ .

(c)  $\int_0^1 x^2 - 1 dx \leq \frac{\sqrt{7}}{3}$ .

(d)  $\left(\int_1^2 x^2 dx\right) \left(\int_1^2 \frac{1}{x^2} dx\right) \geq 4$ .