MATH 3B Final practice problems

1. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^4 v'(t)\, dt$ represent?

2. Compute the following:

   (a) $\displaystyle \int e^{-x} \cos(2x)\, dx$

   (b) $\displaystyle \int \frac{1+x^2}{\sqrt{3x+x^3}}\, dx$

   (c) $\displaystyle \int \cos(\sqrt{x})\, dx$

   (d) $\displaystyle \frac{d}{dx} \int_x^{x^2} t^3\, dt$

   (e) $\displaystyle \int \frac{(ln(x))^3}{x}\, dx$

   (f) $\displaystyle \int \frac{x^3 - \sqrt{x}}{x^2}\, dx$

   (g) $\displaystyle \int \sin^3(x)\cos^2(x)\, dx$

   (h) $\displaystyle \int_0^1 \frac{4}{4x-1}\, dx$

   (i) $\displaystyle \int \frac{1}{x^2-1}\, dx$

3. Consider the region $R = \{(x,y) : x \geq 1, 0 \leq y \leq 1/x\}$.

   (a) Show that the region R has infinite area.

   (b) Find the volume of the solid obtained by rotating R about the x-axis. It is finite, which is kinda crazy given part (a), huh?

4. Find the average value of $f(x) = \frac{x}{x-6}$ for $0 \leq x \leq 2$.

5. Find the work required to lift a chain lying on the ground to a height of 10 feet if the chain weighs 20 pounds and is 10 feet long.

6. Find the length of the arc $y = x^{3/2}$ for $0 \leq x \leq 4$.

7. Evaluate $\displaystyle \int_0^\infty \frac{1}{x-2}\, dx$ or show that the integral diverges.

8. Find the volume of the solid obtained by rotating the region bounded by the curves $y = 2x$ and $y = x^2$ about the x-axis.

9. Let $f(x) = x^2$. Approximate $\displaystyle \int_0^4 x^2\, dx$ using a right hand Riemann sum and 4 subintervals.

10. Find the arc length function $s(x)$ from the point $(1,0)$ for the function $f(x) = \int_1^x \sqrt{t^3 - 1}\, dt$.

11. Hydrostatic force and center of mass: you just did a homework on it :)