

Quiz (a)

Solve the following system of linear equations:

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= -2 \\ -x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - x_2 + 3x_3 &= -7 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 3 & -7 \end{array} \right] \xrightarrow{\substack{R_1 + R_2 \\ R_3 - 2R_1}} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & 3 & -3 & -3 \end{array} \right]$$

$$\xrightarrow{\substack{-R_2 \\ 3R_2 + R_3}} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ (REF)}$$

$$\xrightarrow{R_1 + 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ (RREF)}$$

x_1 and x_2 are leading variables and x_3 is a free variable.

$$\begin{array}{l} x_1 + x_3 = -4 \\ x_2 - x_3 = -1 \\ x_3 = r, \text{ where } r \text{ is any real number} \end{array} \longrightarrow \begin{array}{l} x_1 = -4 - x_3 = -4 - r \\ x_2 = -1 + x_3 = -1 + r \\ x_3 = r \text{ where } r \text{ is any real number} \end{array}$$

$$\text{or } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \text{ where } r \text{ is any real number}$$

is the solution. Since $x_3 = r$ can be any real number, there are infinitely many solutions.