

Quiz 1(b)

Solve the following system of linear equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -2 \\ -x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - x_2 + 3x_3 &= -7 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 3 & -7 \end{array} \right] \xrightarrow{\substack{R_1+R_2 \\ R_3-2R_1}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 0 & -5 & -3 & -3 \end{array} \right]$$

$$\xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -5 & -3 & -3 \end{array} \right] \xrightarrow{R_3+5R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -8 & -8 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{8}R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right] \text{ (REF)}$$

$$\xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{\substack{R_1-5R_3 \\ R_2+R_3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] \text{ (RREF)}$$

Note that x_1, x_2, x_3 are leading variables (no free variables).

$$\begin{array}{r} x_1 \\ x_2 \\ x_3 \end{array} = \begin{array}{l} -5 \\ 0 \\ 1 \end{array} \text{ or } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}$$

is the unique solution to the system.