Quiz 2

NAME:
PERM NO.:
SECTION (CIRCLE ONE): 8AM 5PM 6PM 7PM

Note: To be completely rigorous we should note that $\mathbb{W}$ is a subset of the corresponding vector space, and that $\mathbb{W}$ is nonempty (ie, $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \in \mathbb{W}$ for #1, and $p(x) \overset{def}{=} 0 \in \mathbb{W}$ for #2).

1. Let $\mathbb{W}$ be the set of all matrices of the form

$$\begin{pmatrix} 2 & a_{12} \\ 0 & a_{22} \end{pmatrix}.$$

Is $\mathbb{W}$ a subspace of $\mathbb{M}_{22}$ (the vector space of all 2×2 matrices)? If so, show that it is. If not, show that a vector space property is violated.

We are given that $\mathbb{M}_{22}$ is a vector space so we only need to check that: ① If $A, B \in \mathbb{W}$, then $A+B \in \mathbb{W}$.
② If $A \in \mathbb{W}$ and $c \in \mathbb{R}$, then $cA \in \mathbb{W}$.

①. Let $A = \begin{pmatrix} 2 & a_{12} \\ 0 & a_{22} \end{pmatrix} \in \mathbb{W}$, $B = \begin{pmatrix} 2 & b_{12} \\ 0 & b_{22} \end{pmatrix} \in \mathbb{W}$.

Then $A+B = \begin{pmatrix} 2 & a_{12} \\ 0 & a_{22} \end{pmatrix} + \begin{pmatrix} 2 & b_{12} \\ 0 & b_{22} \end{pmatrix} = \begin{pmatrix} 4 & a_{12}+b_{12} \\ 0 & a_{22}+b_{22} \end{pmatrix}$

$(A+B) \notin \mathbb{W}$ as $(A+B)_{11} = 4 \neq 2$.

② Note that $2\begin{pmatrix} 2 & a_{12} \\ 0 & a_{22} \end{pmatrix} = \begin{pmatrix} 4 & a_{12} \\ 0 & a_{22} \end{pmatrix} \notin \mathbb{W}$ for the same reason as ①.

So both conditions fail, therefore $\mathbb{W}$ is <u>not</u> a subspace of $\mathbb{M}_{22}$. (You only need to show that one condition failed)

2. Let $\mathbb{P}_3$ be the vector space of all polynomials of degree less than or equal to 3, and let $\mathbb{W} = \{p(x) \in \mathbb{P}_3 : p(0) = 0\}$. Is $\mathbb{W}$ a subspace of $\mathbb{P}_3$? If so, show that it is. If not, show that a vector space property is violated.

We are given $\mathbb{P}_3$ is a vector space, so we only need to check conditions ① and ② as in problem 1.

① Let $p_1(x), p_2(x) \in \mathbb{W}$. Then since $p_1(x), p_2(x) \in \mathbb{P}_3$,
$p_1(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, $p_2(x) = b_0 + b_1x + b_2x^2 + b_3x^3$ where $a_i, b_i \in \mathbb{R}$.
Then $(p_1+p_2)(x) \overset{def}{=} p_1(x) + p_2(x) = (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$.
Since $a_i + b_i \in \mathbb{R}$, $(p_1+p_2)(x) \in \mathbb{P}_3$.
We also know that $p_1(0) = 0$, $p_2(0) = 0$. Then $(p_1+p_2)(0) \overset{def}{=} p_1(0) + p_2(0) = 0+0 = 0$.
Therefore $(p_1+p_2)(x) \in \mathbb{W}$.

② Let $p(x) \in \mathbb{W}$, $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, where $a_i \in \mathbb{R}$. Then for all $c \in \mathbb{R}$,
$(cp)(x) \overset{def}{=} cp(x) = ca_0 + ca_1x + ca_2x^2 + ca_3x^3$, and $ca_i \in \mathbb{R}$, so $(cp)(x) \in \mathbb{P}_3$.
Since $p(0) = 0$, we have that $(cp)(0) = cp(0) = c \cdot 0 = 0$, so $(cp)(x) \in \mathbb{W}$.
Then since ① and ② hold, $\mathbb{W}$ <u>is</u> a subspace of $\mathbb{M}_{22}$.