Quiz 3

NAME:
PERM NO.:
SECTION (CIRCLE ONE): 8AM 5PM 6PM 7PM

1. Consider the set of vectors

$$\{[1,0],[0,1],[1,1]\}$$

in the vector space $\mathbb{R}^2$. Justify your answers to the following:
(a) Are these vectors linearly independent?
(b) Do they span $\mathbb{R}^2$?
(c) Do they form a basis for $\mathbb{R}^2$?

(a) No. Notice we can write the third as a linear combination of the first two:

$$\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}.$$

(b). Yes. Notice that $\operatorname{span}\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$

$$= \left\{c_1\begin{bmatrix}1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\end{bmatrix} : c_1, c_2 \in \mathbb{R}\right\}$$

$$= \left\{\begin{bmatrix}c_1\\c_2\end{bmatrix} : c_1, c_2 \in \mathbb{R}\right\}$$

$$= \mathbb{R}^2.$$

(c). No. (a) and (b) must hold for $\{[1,0],[0,1],[1,1]\}$ to form a basis for $\mathbb{R}^2$.

(you can check that $\{[1,0],[0,1]\}$ is a basis for $\mathbb{R}^2$.

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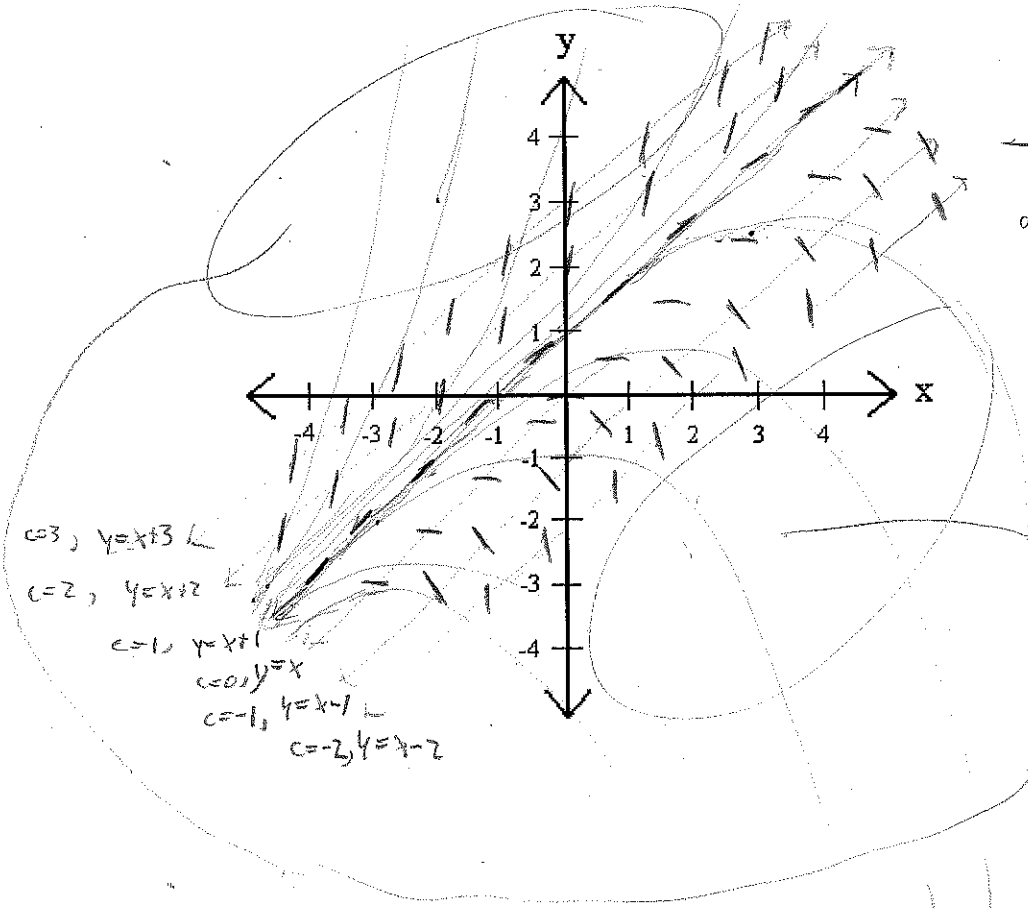
1. For the differential equation

$$y' = y - x,$$

lightly sketch several isoclines (start with $c = 0, \pm 1, \pm 2$), and use these to sketch a direction field with some sample solutions drawn darker.



$y' = y - x = c$

$\longrightarrow$ $y = x + c$, so the isoclines are lines with slope = 1 and y-intercept at c. Solutions to our DE will have slope = c when crossing the line $y = x + c$.

After beginning to sketch a direction field using the isoclines, notice that the slope continues to decrease here (from left to right), and continues to increase here (from right to left).

This helps us sketch some solutions to our DE.

Common mistake: We can never have a solution like this: