Quiz ~~4~~ 5

NAME:
PERM NO.:
SECTION (CIRCLE ONE): 8AM 5PM 6PM 7PM

1. Use the integrating factor method to find the solution to the following IVP. You may use the fact that $\sin(2x) = 2\sin(x)\cos(x)$.

$$\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1, \qquad y(\tfrac{\pi}{4}) = 3\sqrt{2}, \qquad (0 \le x < \tfrac{\pi}{2}).$$

Start by dividing by $\cos(x)$ to get the DE in the form $y' + p(x)y = f(x)$:

$\rightarrow$ $y' + \tan(x)y = 2\cos^2(x)\sin(x) - \sec(x)$.

Now we can find the integrating factor:

$\mu(t) = e^{\int p(x)\,dx} = e^{\int \tan(x)\,dx} = e^{\ln(\sec x)} = \sec(x)$

If you forgot $\int \tan(x)\,dx$, write $\int \frac{\sin x}{\cos x}\,dx$; $u = \cos x$, $du = -\sin x\,dx$

$-\int \frac{1}{u}\,du = -\ln\cos x = \ln(\cos x)^{-1} = \ln\sec x$.

Now we multiply the integrating factor $\sec(x)$ through the equation in the form $y' + p(x)y = f(x)$:

$\sec(x)y' + \sec(x)\tan(x)y = 2\sec(x)\cos^2(x)\sin(x) - \sec^2(x)$

$\rightarrow$ $(\sec(x)y)' = 2\cos(x)\sin(x) - \sec^2 x$ (double check the LHS $= (\mu(x)y)'$ using product rule)

$\rightarrow$ $(\sec(x)y)' = \sin(2x) - \sec^2 x$, using the hint.

$\rightarrow$ $\int (\sec(x)y)'\,dx = \int \sin(2x) - \sec^2(x)\,dx$

$\rightarrow$ $\sec(x)y = -\frac{1}{2}\cos(2x) - \tan(x) + c$.

Now divide by $\sec(x)$ (multiply by $\cos(x)$) to solve for y:

$y = -\frac{1}{2}\cos(x)\cos(2x) - \sin(x) + C\cos(x)$. This is the general solution.

Use $y(\frac{\pi}{4}) = 3\sqrt{2}$ to find c:

$3\sqrt{2} = -\frac{1}{2}\cos(\frac{\pi}{4})\cos(\frac{\pi}{2}) - \sin(\frac{\pi}{4}) + C\cos(\frac{\pi}{4})$

$\rightarrow 3\sqrt{2} = \cancel{-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot 0} - \frac{\sqrt{2}}{2} + c\,\frac{\sqrt{2}}{2}$

$\rightarrow \frac{6\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = c\frac{\sqrt{2}}{2} \rightarrow c = 7 \longrightarrow \boxed{y = -\frac{1}{2}\cos(x)\cos(2x) - \sin(x) + 7\cos(x)}$