Math 4A Concept Check

Post-midterm 2

Definitions

Dimension, rank, nullity, eigenvalue, eigenvector, eigenspace, characteristic equation, (algebraic) multiplicity, similar matrices, diagonalizable, inner/dot product, length/norm, orthogonal/perpendicular, unit vector, normalize, least-squares solution, orthogonal set, orthogonal basis, orthonormal set, orthonormal basis

You should know:

- How to determine the dimension of Nul A and Col A (the nullity and rank of A).
- The rank-nullity theorem.
- Statements about Null A, Col A, nullity, and rank to add to the core theorem (invertible matrix theorem).
- How to find eigenvalues and corresponding eigenvectors and eigenspaces.
- What the eigenvalues of a triangular matrix are.
- Eigenvectors that correspond to distinct eigenvalues are linearly independent.
- How to check if a given vector is an eigenvector for A.
- A necessary and sufficent condition for A to be diagonalizable.
- How to diagonalize a matrix, if possible.
- A formula for A^k if A is diagonalizable.
- How to determine when a given 2×2 matrix has 0, 1, or 2 real eigenvalues.
- A square matrix is invertible if and only if what number is not an eigenvalue?
- How to find complex eigenvalues.
- How to compute the inner/dot product of two vectors.
- How to check if two vectors are orthogonal.
- How to compute the length of a vector or the distance between two vectors.
- How to find a unit vector in the same direction as a given vector.
- How to find the least-squares solution set to a system of equations.
- How to find the orthogonal complement of a vector.
- The relation between orthogonal complement and null space. What is the dimension of the orthogonal complement of a vector?
- How to find linearly independent vectors orthogonal to a given vector.
- How to find the coordinates of a vector that is in the subspace spanned by an orthogonal set of vectors (same as finding the coordinate vector for such a vector or writing such a vector as a linear combination of the vectors in the orthogonal basis given).

Post-midterm 1, pre-midterm 2

Definitions

Linear transformation, domain, codomain, image, linear combination, span, linear independent/dependent, dependence relation, one-to-one, onto, inverse matrix, identity matrix, (non-)invertible, (non-)singular, transpose matrix, (skew-)symmetric, diagonal matrix, upper/lower triangular matrix, determinant, cofactor, vector space, (vector) subspace, "closed under", kernel, range, null space, column space, basis, standard basis, $\mathbb{P}_n$, $C(I)$, $C^n(I)$, $M_n(\mathbb{R})$, the coordinates of a vector relative to a basis, change-of-coordinate matrix

You should know:

- How to check if vectors are linearly independent.
- How to write a linear transformation as $T(\mathbf{x}) = A\mathbf{x}$.
- How to check if T is one-to-one and/or onto.
- How to find A^{-1}.
- when a 2×2 matrix is invertible and the formula to find A^{-1}
- How to solve a system of equations given A^{-1}.
- How to determine if A is invertible.
- A list of equivalent statements to A being invertible.
- How to compute a determinant using cofactor expansion.
- How to compute a determinant using row operations and REF.
- The effect of row (or column) operations on determinants.
- A is invertible if and only if its determinant is what?
- Determinants of products, inverses, transposes.
- Geometric interpretations of determinants.
- Examples of common vector spaces.
- How to show whether or not a set is a vector space/subspace.
- The connection between span and subspaces.
- The relations between kernel, null space, and one-to-one.
- The relations between range, column space, and onto.
- How to check if a vector is in the kernel/null space or range/column space.
- Kernels/null spaces and ranges/column spaces are subspaces (of what vector spaces?).
- How to find a basis for the kernel/null space and range/column space.
- How to determine if a set of vectors is a basis of a vector space.
- Standard bases of common vector spaces.
- How to find the kernel of of a transformation, say $T : \mathbb{P}_2 \to \mathbb{P}_2$.
- How to write a non-trivial linear dependence relation given a set of linearly dependent vectors.
- How to check if polynomials are linearly independent.
- How to check if a set of polynomials span the whole space.
- How to check if a polynomial is in the span of other polynomials.
- How to find the change-of-coordinate matrix.
- How to find coordinate vectors relative to different bases.

Pre-midterm 1

Definitions

Linear equation, leading entry, REF, RREF, pivot, pivot column, free variable, linear combination, span, linear independent/dependent, (non)trivial solution, dependence relation, (non)homogeneous, linear transformation

You should know:

- The conditions for a matrix to be in REF or RREF, and how to find the REF or RREF of a matrix.
- How to solve a system of equations/vector equation/matrix equation.

- How to determine the values of some coefficient (like h or k) that: make a system of equations consistent/inconsistent, vectors span the whole space or not, make vectors linearly independent or not, make a vector be in the range of a linear transformation or not, etc.
- How to describe a set of infinitely many solutions in parametric vector form.
- How to describe a set of infinitely many solutions using span notation.
- Whether a system of equations has one solution, infinitely many solutions, or no solutions by considering pivot columns of its augmented matrix.
- Whether vectors are linearly independent based or span the whole space based on pivots in columns or rows.
- How to determine if a vector is in the span of a set of vectors.
- Geometric interpretations of linear combinations, span, and linear indendence/dependence.
- The parallegram rule for vector addition.
- Linearity properties of the matrix-vector product $A\mathbf{x}$.
- Matrix operations - multiplication, addition, scalar multiplication, transpose.
- If $m < n$, can a set of m vectors in $\mathbb{R}^n$ span $\mathbb{R}^n$?
- If $m > n$, can a set of m vectors in $\mathbb{R}^n$ be linearly independent?
- Equivalent statements the matrix equation $A\mathbf{x} = \mathbf{b}$, each $\mathbf{b} \in \mathbb{R}^m$ being a linear combination of the column vectors of A, the column vectors of A spannning $\mathbb{R}^m$, A having a pivot in every **row**.