Integration concepts

	Intervals (3B)	Curves	Regions	Surfaces	Solids
Integrals of real-valued functions	$\int_I f(t)\ \mathrm{d}t$	$\int_{\mathbf{c}} f\ \mathrm{d}s$ $= \int_a^b f(\mathbf{c}(t))\lvert\lvert\mathbf{c}'(t)\rvert\rvert\ \mathrm{d}t$	$\iint_D f(x,y)\ \mathrm{d}A$	$\int_S f\ \mathrm{d}S$ $= \iint_D f(\mathbf{r}(u,v))\lvert\lvert\mathbf{r}_u \times \mathbf{r}_v\rvert\rvert\ \mathrm{d}A$	$\iiint_E f(x,y,z)\ \mathrm{d}V$
Integrals of vector-valued functions		$\int_{\mathbf{c}} \mathbf{F}\cdot\ \mathrm{d}\mathbf{s}$ $= \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\ \mathrm{d}t$		$\iint_S \mathbf{F}\cdot\ \mathrm{d}\mathbf{S}$ $= \iint_D \mathbf{F}(\mathbf{r}(u,v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v)\ \mathrm{d}A$	
Physics		Work $= \int_{\mathbf{c}} \mathbf{F}\cdot\ \mathrm{d}\mathbf{s}$		Flux $= \iint_S \mathbf{F}\cdot\ \mathrm{d}\mathbf{S}$	
Length/area/volume	Length of interval $= \int_I f(t)\mathrm{d}t$	Arclength $= \int_{\mathbf{c}} \mathrm{d}s = \int_a^b \lvert\lvert\mathbf{c}'(t)\rvert\rvert\ \mathrm{d}t$	Area of D $= \iint_D \mathrm{d}A$	Surface area of S $= \int_S \mathrm{d}S = \iint_D \lvert\lvert\mathbf{r}_u \times \mathbf{r}_v\rvert\rvert\mathrm{d}A$	Volume of E $= \iiint_E \mathrm{d}V$
Fundamental Theorem of Calculus	$\int_a^b f'(t)\ \mathrm{d}t$ $= f(b) - f(a)$	$\int_{\mathbf{c}} \nabla f\cdot\ \mathrm{d}\mathbf{s}$ $= f(\mathbf{c}(b)) - f(\mathbf{c}(a))$	$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\ \mathrm{d}A$ $= \int_{\partial D} \mathbf{F}\cdot\ \mathrm{d}\mathbf{s}$ (Green's)	$\iint_S (\nabla \times \mathbf{F})\cdot\ \mathrm{d}\mathbf{S}$ $= \int_{\partial S} \mathbf{F}\cdot\ \mathrm{d}\mathbf{s}$ (Stokes')	$\iiint_E (\nabla \cdot \mathbf{F})\ \mathrm{d}V$ $= \iint_{\partial E} \mathbf{F}\cdot\ \mathrm{d}\mathbf{S}$ (Div Thm)
Change of variables	$\int_I f(x)\ \mathrm{d}x$ $= \int_{I^*} f(x(u)) \left\lvert\frac{\mathrm{d}x}{\mathrm{d}u}\right\rvert\ \mathrm{d}u$		$\iint_D f(x,y)\ \mathrm{d}A$ $= \iint_{D^*} f(x(u,v),y(u,v))$ $\left\lvert\frac{\partial(x,y)}{\partial(u,v)}\right\rvert\ \mathrm{d}A^*$		$\iiint_E f(x,y,z)\ \mathrm{d}V$ $= \iint_{E^*} f(x(u,v,w),y(u,v,w),z(u,v,w))$ $\left\lvert\frac{\partial(x,y,z)}{\partial(u,v,w)}\right\rvert\ \mathrm{d}V^*$
Dependence on parametrization (real-valued functions)		$\int_{\mathbf{c}} f\ \mathrm{d}s$ $= \int_{\mathbf{c}^*} f\ \mathrm{d}s^*$		$\iint_D f(\mathbf{r}(u,v))\lvert\lvert\mathbf{r}_u \times \mathbf{r}_v\rvert\rvert\ \mathrm{d}A$ $= \iint_{D^*} f(\mathbf{r}^*(u^*,v^*))\lvert\lvert\mathbf{r}^*_{u^*} \times \mathbf{r}^*_{v^*}\rvert\rvert\ \mathrm{d}A^*$	
Dependence on parametrization (vector-valued functions)		$\int_{\mathbf{c}} \mathbf{F}\cdot\ \mathrm{d}\mathbf{s}$ $= \pm\int_{\mathbf{c}^*} \mathbf{F}\cdot\ \mathrm{d}\mathbf{s}^*$		$\iint_D \mathbf{F}(\mathbf{r}(u,v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v)\ \mathrm{d}A$ $= \pm\iint_{D^*} \mathbf{F}(\mathbf{r}^*(u^*,v^*)) \cdot (\mathbf{r}^*_{u^*} \times \mathbf{r}^*_{v^*})\ \mathrm{d}A^*$	

Change of variables

In general	Polar coordinates
$\mathrm{d}A = \left\|\frac{\partial(x,y)}{\partial(u,v)}\right\| \mathrm{d}A^*$	$\mathrm{d}A = r\ \mathrm{d}r\ \mathrm{d}\theta$
$x = x(u,v)$ $y = y(u,v)$	$x = r\ \cos\ \theta$ $y = r\ \sin\ \theta$ $x^2 + y^2 = r^2$

In general	Cylindrical coordinates	Spherical coordinates
$\mathrm{d}V = \left\|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right\| \mathrm{d}V^*$	$\mathrm{d}V = r\ \mathrm{d}z\ \mathrm{d}r\ \mathrm{d}\theta$	$\mathrm{d}V = \rho^2\ \sin\ \phi\ \mathrm{d}\rho\ \mathrm{d}\theta\ \mathrm{d}\phi$
$x = x(u,v,w)$ $y = y(u,v,w)$ $z = z(u,v,w)$	$x = r\ \cos\ \theta$ $y = r\ \sin\ \theta$ $z = z$ $x^2 + y^2 = r^2$	$x = \rho\ \sin\ \phi\ \cos\ \theta$ $y = \rho\ \sin\ \phi\ \sin\ \theta$ $z = \rho\ \cos\ \phi$ $x^2 + y^2 + z^2 = \rho^2$