The **simple harmonic oscillator** equation is

$$mx'' + bx' + kx = f(t); \qquad m > 0, k > 0, b \geq 0$$

If $b = 0$, the motion is **undamped**, and if $b > 0$ it is **damped**.

If $f(t) = 0$, the equation is homogeneous, and the motion is **unforced, undriven, or free**. Otherwise the motion is **forced or driven**.

The solutions to the undamped unforced oscillator

$$mx'' + kx = 0$$

are

$$x(t) = c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t); \qquad \omega_0 = \sqrt{k/m}$$

ω_0 is called the **circular frequency**, measured in radians per second.
$f_0 = \omega_0/2\pi$ is the **natural frequency**, measured in oscillations per second.

Alternate forms for solutions are

$$x(t) = R\,\cos(\omega_0 t - \delta) = R\,\cos(\omega_0(t - \delta/\omega_0))$$

or

$$x(t) = R\,\sin(\omega_0 t - \theta) = R\,\sin(\omega_0(t - \theta/\omega_0))$$

These help us graph solutions as sine or cosine waves shifted right δ/ω_0 units.

To convert from one form to the other use

$$R = \sqrt{c_1^2 + c_2^2},\ \tan(\delta) = c_2/c_1,\ \tan(\theta) = -c_1/c_2, c_1 = R\,\cos(\delta) = -R\,\sin(\theta), c_2 = R\,\sin(\delta) = R\,\cos(\theta)$$

R is called the **amplitude** and δ is called the **phase angle**.

Recall that weight is given by $w = mg$ and so we use $m = w/g$ to find m if necessary.

To find the spring constant k use $k = \frac{F}{L} = \frac{mg}{L} = \frac{w}{L}$ where L is the length the spring is stretched.

A system is called:

- overdamped if the damping constant $b > 2\sqrt{mk}$. The system will return (exponentially decay) to equilibrium without oscillating.

- critically damped if the damping constant $b = 2\sqrt{mk}$. The system will return to equilibrium as quickly as possible without oscillating.

- underdamped if the damping constant $0 < b < 2\sqrt{mk}$. The system will oscillate (at reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero.

- undamped if the damping constant $b = 0$. The system will oscillate at its natural resonant frequency (ω_0).