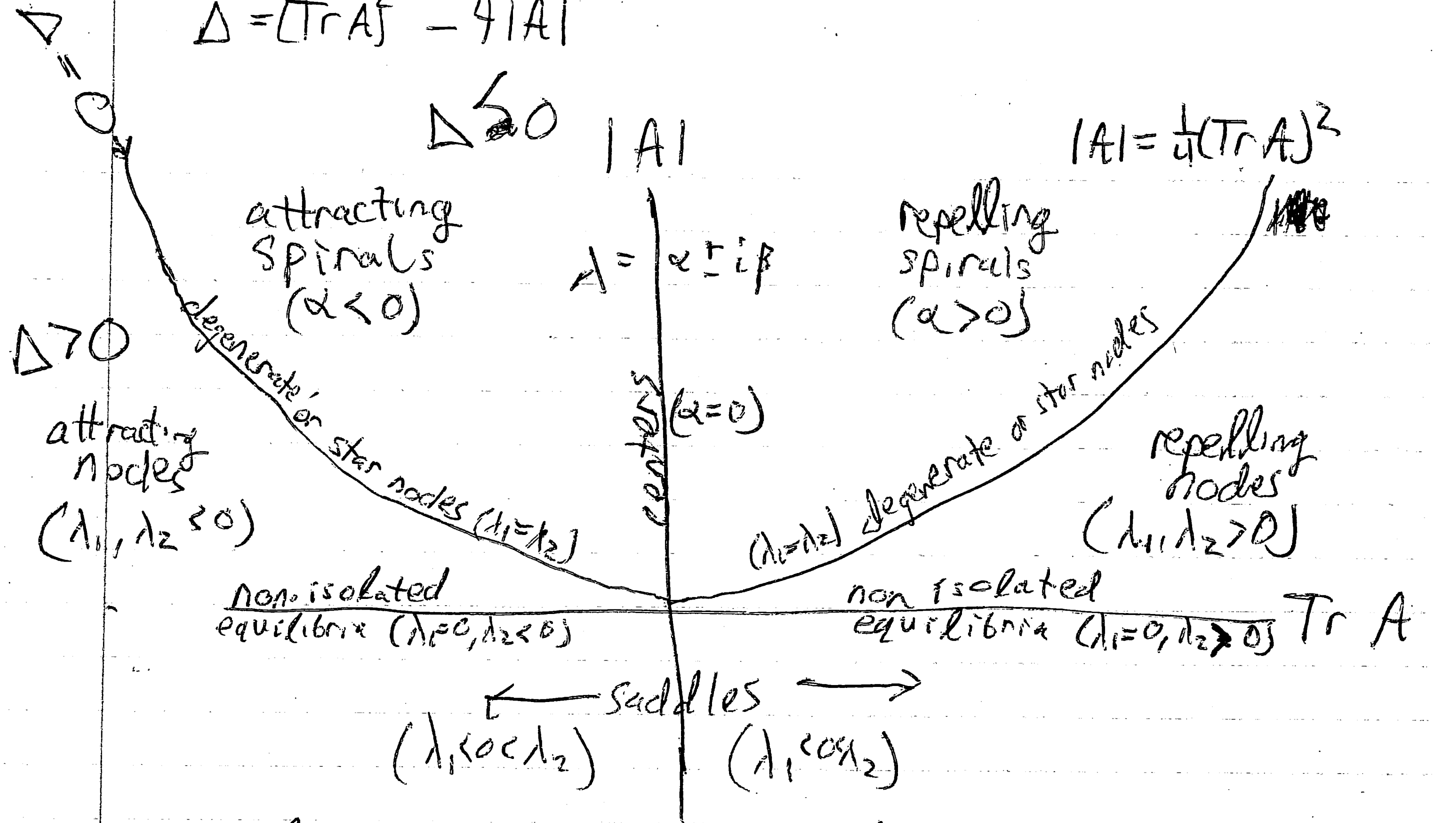
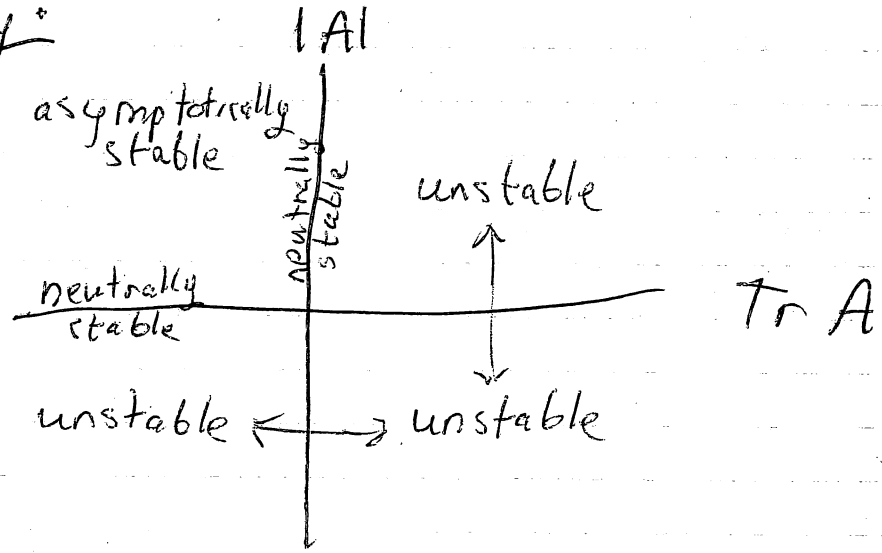
$\Delta = [\mathrm{Tr}\,A]^2 - 4|A|$



Borderline cases: Degenerate node : 1 e-vector
Star node : 2 e-vectors

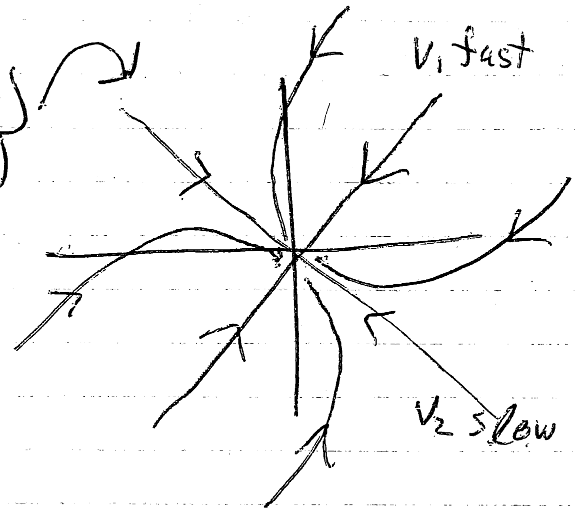
Stability:



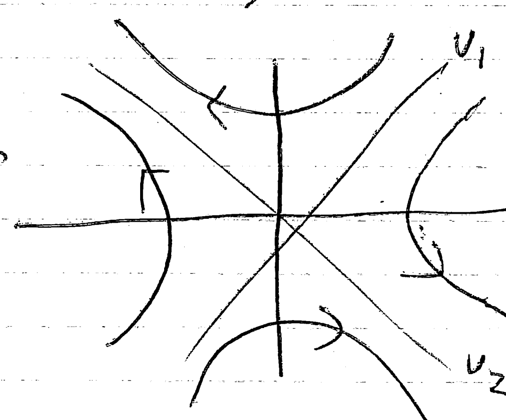
Sketching Phase Portraits

Real roots

- Attracting node ($\lambda_1 < \lambda_2 < 0$)
  (trajectories parallel to v_1 fast)
  (asymptotically stable)
- Repelling node ($0 < \lambda_2 < \lambda_1$)
  just switch directions →
  (unstable)



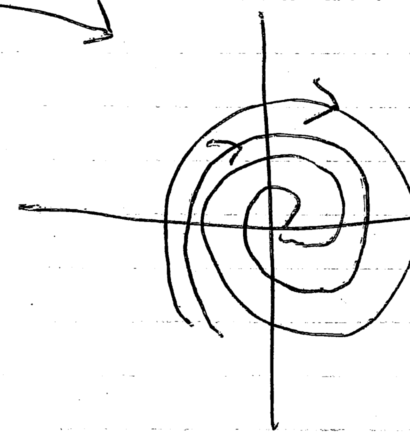
- Saddle ($\lambda_1 < 0 < \lambda_2$)
  ($e^{\lambda_1 t} \to 0$, $e^{\lambda_2 t} \to \infty$, so trajectories approach equilibrium along v_1 then become parallel to v_2)
  (unstable)



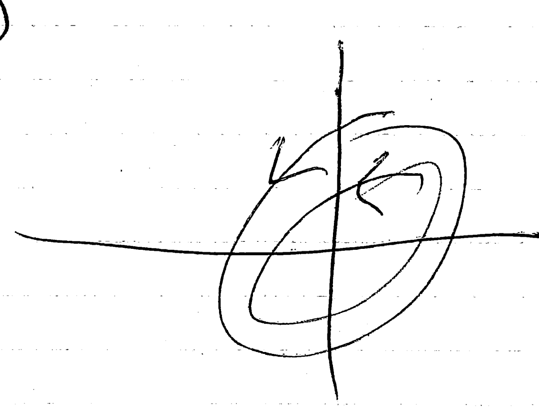
Complex Roots

$\lambda_{1,2} = \alpha \pm i\beta$

- Attracting spiral ($\alpha < 0$)
  (asymptotically stable)
- Repelling spiral ($\alpha > 0$)
  just switch directions →
  (unstable)



- Center ($\alpha = 0$ so no e terms, just sin, cos)
  (neutrally stable)



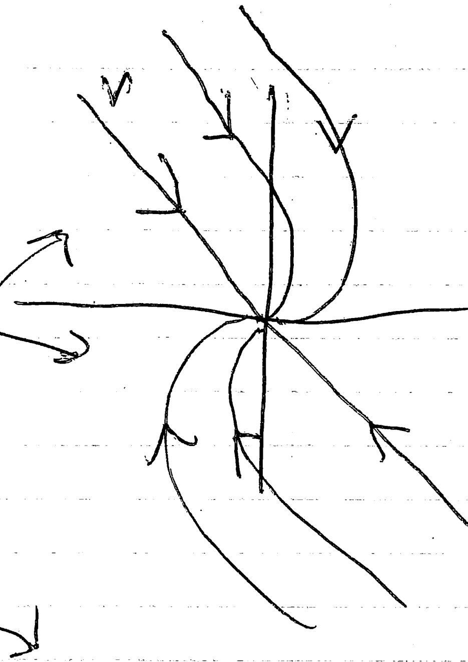
Borderline Cases

Repeated real roots ($\Delta = 0$)

We either get:

- Degenerate node (1 lin. indep. e-vector)
  - $\lambda < 0$: asymptotically stable
  - $\lambda > 0$: unstable (switch arrows)

(trajectories become parallel to $\vec{v}$)

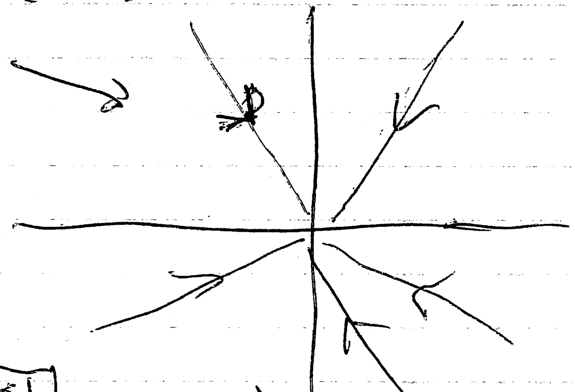


or

- Star node (2 lin. indep. e-vectors)

Every vector is an e-vector

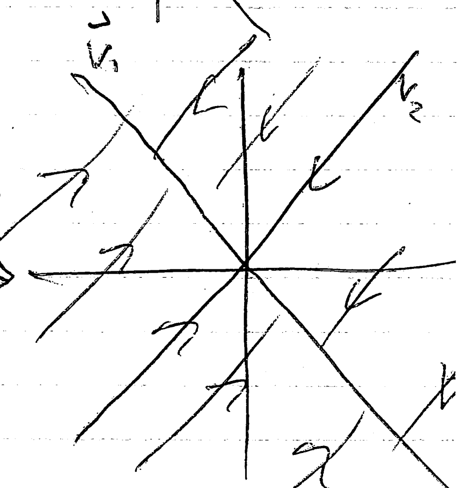
  - $\lambda < 0$: asymptotically stable
  - $\lambda > 0$: unstable (switch arrows)



One or two real roots $= 0$ ($|A| = 0$)

- Get non isolated equilibria
- $\lambda_1 = 0$, and
  - $\lambda_2 < 0$: asympotically stable
  - $\lambda_2 > 0$: unstable (switch)

(line of equilibrium solutions along v_1, trajectories parallel to v_2)



- $\lambda_1 = \lambda_2 = 0$

(1 e-vector v, trajectories parallel to $\vec{v}$)
Use DE system to determine direction)