MAT 116 : Midterm Exam Study Guide

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Remarks

The midterm exam will consist mostly of computations, and it might also contain some short proofs or explainations. Your exam solutions are expected to be written clearly. You may apply theorems (without proof) if they were proved in class or in Brualdi's book; you may (and are encouraged to) apply combinatorial principles and formulas such as Pascal's formula. In particular, you may not use results proved on homework or in the in-class problems: it is possible that an exam question may actually be (part of) a homework problem. If you are not sure about whether or not you can apply a particular result while taking the test, you may ask me.

Practice Problems:

- All of the *problems for understanding* from Hw 1 and 2
- In-class problems: sets 1 through 4 (now available at the class website)

Four basic counting principles

Principles: Addition, multiplication, subtraction, and division principles.

Permutations of sets

Definitions: Permutation of a set, r–element permutation of an n–element set (i.e. r–permutation), ordinary permutation as linear permutation, circular permutation of a finite set, r–element circular permutation of an n–element set (i.e. circular r–permutation).

Formulas:

- The number of r–permutations of an n–element set is

$$P(n,r) = \frac{n!}{(n-r)!}\ .$$

- The number of circular r–permutations of an n–element set is

$$Circ(n,r) = \frac{P(n,r)}{r} = \frac{n!}{r \cdot (n-r)!}\ .$$

Combinations (subsets) of sets

Definitions: Combination (subset) of a set, r–element combination (subset) of an n–element set (i.e. r–combination).

Formulas:

- The number of r–combinations of an n–element set is

$$C(n,r) = \binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{r! \cdot (n-r)!}\ .$$

- Pascal's formula is

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}\ .$$

Permutations of multisets

Definitions: Multiset, types, repetition numbers, r–permutation of a multiset.

Formulas: Let S be a multiset with k types of elements. We usually write this as

$$S = \{n_1 \cdot a_1, \ldots, n_k \cdot a_k\}$$

where the a_i are the *types* and the n_i are their respective (possibly infinite) *repetition numbers*.

- The number of r–permutations of S with all *infinite* (or *sufficiently large*) repetition numbers is k^r.

- The number of permutations of S with all *finite* repetition numbers $n_1, \ldots, n_k$ such that $|S| = n = n_1 + \cdots + n_k$ is
$$\frac{n!}{n_1! \cdots n_k!} .$$

Combinations (subsets) of multisets

Definitions: Multiset, types, submultiset (or combination), repetition numbers, r–combination of a multiset.

Formulas/Techniques: Let S be a multiset with k types of elements. We usually write this as

$$S = \{n_1 \cdot a_1, \ldots, n_k \cdot a_k\}$$

where the a_i are the *types* and the n_i are their respective (possibly infinite) *repetition numbers.*

- The number of r–combinations of S with all *infinite* (or *sufficiently large*) repetition numbers is
$$\binom{r+k-1}{r} = \binom{r+k-1}{k-1} ,$$
and that integer is the same as the number of nonnegative integral solutions of
$$x_1 + \cdots + x_k = r .$$

- When counting the number of integral solutions of
$$x_1 + \cdots + x_k = r$$
with constraints
$$x_1 \geq l_1, \ldots x_k \geq l_k ,$$
change variables: $y_i = x_i - l_i$, then use the standard formula.

Pidgeonhole principle: simple form

Pidgeonhole Principle (Simple Form). *If $n+1$ objects are distributed into n boxes, then at least one box contains two or more of the objects.*

Applications: See Applications 1 through 6 in section 3.1 of the textbook.