MAT 116 In-class Problems (#5)

July 6, 2010

Remark. Just to be clear, when we say "the box contains r objects", we really mean that the box contains *at least* r objects.

Pidgeonhole Principle (Strong Form). *Let $q_1, q_2, \ldots, q_n$ be positive integers. If*

$$q_1 + q_2 + \cdots + q_n - n + 1$$

objects are distributed into n boxes, then the first box contains q_1 objects, or the second box contains q_2 objects, $\ldots$, or the nth box contains q_n objects.

Corollary. *If n and r are positive integers and $n(r-1)+1$ objects are distributed into n boxes, then at least one of the boxes contains r objects.*

Application 7. A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruit that should be put in the basket to guarantee that either there are at least eight apples or at least six bananas or at least nine oranges?

Averaging Principle. *If the average of n nonnegative integers $m_1, m_2, \ldots, m_n$ is at least equal to r, then at least one of the integers $m_1, m_2, \ldots, m_n$ satisfies $m_i \geq r$.*

Application 8. Two disks, one smaller than the other, are each divided into 200 congruent sectors. In the larger disk, 100 of the sectors are chosen arbitrarily and painted red; the other 100 sectors are painted blue. In the smaller disk, each sector is painted either red or blue with no stipulation on the number of red and blue sectors. The small disk is then placed on the larger disk so that their centers coincide. Show that it is possible to align the two disks so that the number of sectors of the small disk whose color matches the corresponding sector of the large disk is at least 100.