MAT 145 : Homework Hints

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Last Updated: May 11, 2009

Homework 4 Hints

Problems from Crossley's Book

5.1 The simplest function to use would be an *affine function* (i.e. a function of the form $f(x) = mx+b$, where m and b are constants). To find a formula for $f(x)$, first construct the *appropriate* straight line in $\mathbb{R}^2$, then derive the equation $y = mx + b$ for this line. After finding $f(x)$, the inverse function should be extremely easy to calculate.

5.5 For stereographic projection of $\mathbb{S}^1$: Let (x, y) be a point in $\mathbb{S}^1 - \{(0, 1)\}$. Derive an equation for the line passing through $(0, 1)$ and (x, y); you might want to use variables like x_1, x_2 for this equation. Then calculate the *appropriate* intersection point of this line with the projection line $\{(x_1, -1) : x_1 \in \mathbb{R}\}$. Draw a diagram of $\mathbb{S}^1$ and the projection line in $\mathbb{R}^2$ to understand the geometry of this problem.

For stereographic projection of $\mathbb{S}^2$: Let (x, y, z) be a point in $\mathbb{S}^2 - \{(0, 0, 1)\}$. Derive parametric equations for the line passing through $(0, 0, 1)$ and (x, y, z); you might want to use variables like x_1, x_2, x_3 for the parametric equations. Then calculate the *appropriate* intersection point of this line with the projection plane $\{(x_1, x_2, -1) : x_1, x_2 \in \mathbb{R}\}$. Draw a diagram of $\mathbb{S}^2$ and the projection plane in $\mathbb{R}^3$ to understand the geometry of this problem.

Problems from Hatcher's Notes

3. Given an open covering $\{O_\alpha\}$ of X, use the complement of one of the O_α's to construct a finite subcovering.

6. It is straightforward to show that $A \cup B$ is compact.

   To show that $A \cap B$ is compact (now assuming that X is Hausdorff), use the fact that $A \cap B$ lies in the compact Hausdorff space A (or B), and use the relationships between *closed sets* and *compact sets* in a compact Hausdorff space. See propositions on pages 33 and 35 of Hatcher's notes; these propositions were also discussed in class. Working with subspaces might be tricky, so read the lemma (and its proof) on page 11 of Hatcher's notes.