MAT 145 : Homework Hints

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Homework 5 Hints

Problems from Crossley's Book

5.9 To show that $S \times T$ is *connected* only if S and T are both *connected*, use the appropriate projection functions. The same hint applies when we replace *"connected"* with *"compact"*.

Problems from Hatcher's Notes

7. Use the fact that $(x, y) \in \overline{A \times B}$ if and only if every basis-element neighborhood $U \times V$ of (x, y) (where U is open in X, and V is open in Y) intersects $A \times B$. You may also use the fact that $(U \times V) \cap (A \times B) = (U \cap A) \times (V \cap B)$.

   Use the fact that $(x, y) \in \text{int}(A \times B)$ if and only if there exists a basis-element neighborhood $U \times V$ of (x, y) (where U is open in X, and V is open in Y) such that $U \times V \subset A \times B$. You may also use the fact that $(U \times V) \cap (A \times B) = (U \cap A) \times (V \cap B)$.

   You will see that using basis elements is very helpful when working with product spaces.

14. In this problem, you want to prove that the distance function $d : X \times X \to \mathbb{R}$ is continuous with respect to the product topology on the domain $X \times X$ and the usual topology on the co-domain $\mathbb{R}$. It suffices to show that for every open interval (i.e. basis element) $(\alpha, \beta) \subset \mathbb{R}$, the pre-image $d^{-1}(\alpha, \beta)$ is open in the product $X \times X$. Here is the simplest way to show that $d^{-1}(\alpha, \beta)$ is open: show that for every $(x, x') \in d^{-1}(\alpha, \beta)$, there exists a basis element $V \times V'$ (where V

is open in X, and V' is open in X) such that $(x, x') \in V \times V' \subset d^{-1}(\alpha, \beta)$; this immediately implies that $\beta > 0$.

Suggestion: You want to define $V = B_r(x)$ and $V' = B_r(x')$ for a sufficiently small $r > 0$; the value of r depends on the real numbers α, β, and $d(x, x')$ (plot these numbers in $\mathbb{R}$ to determine r). The triangle inequality will be very useful in this problem.