Problem 1	Problem 2	Problem 3		Grade

Introduction to Numerical Analysis - Math 104B   Summer 2011   Midterm exam 8/18/2011

Name:

PROBLEM 1 (20 Points) Prove the following:

1. (10 Points) For any vector $x \in \mathbb{R}^n$

$$\frac{\|x\|_1}{n} \le \|x\|_\infty \le \|x\|_1$$

2. (10 Points) For any matrix $A \in \mathbb{R}^{n\times n}$

$$\frac{\|A\|_1}{n} \le \|A\|_\infty \le n\|A\|_1$$

PROBLEM 2 (20 Points)

1. (10 Points) Compute, if possible, the LDL^T decomposition, (L unit lower triangular and D diagonal and invertible) with no pivoting, of the symmetric matrix

$$A = \begin{bmatrix} 2 & -4 & 4 & 0 \\ -4 & 11 & -8 & -6 \\ 4 & -8 & 9 & -2 \\ 0 & -6 & -2 & 18 \end{bmatrix}$$

2. (10 Points) Is A positive definite? Why? If the answer was "yes", compute the Cholesky factor of A, that is, a lower triangular matrix R such that $A = RR^T$

PROBLEM 3 (20 Points)

1. (10 Points) Find $\alpha > 0$ and $\beta > 0$ such that the matrix

$$A = \begin{bmatrix} 3 & 2 & \beta \\ \alpha & 5 & \beta \\ 2 & 1 & \alpha \end{bmatrix}$$

is strictly diagonally dominant.

2. (10 Points) Suppose A and B are symmetric positive definite matrices.

   1. Is $A + B$ positive definite?
   2. Is $A - B$ positive definite?