Homework 2 – Math 104B, Summer 2011

Due on Thursday, August 18th, 2011

Section 7.1: 3.d, 4.d, 5.a, 7, 9, and 11.

Programming Problem:

1. Write a program that implements the Cholesky factorization seen in class, $A = RR^T$. The input should be a real square symmetric matrix A and the outputs should be two:

   (a) A flag variable: $f = 1$, if the matrix is positive definite, and $f = 0$, if it doesn't.

   (b) R a lower triangular matrix.

   If $f = 0$, the contents of the output variable R does not matter.

2. Use the Cholesky program and the triangular solver in homework assignement 1 to write a program that solves the linear system $Ax = b$ when A is symmetric positive definite.

3. To test your routines, as in homework assignement 1, consider the $n \times n$ matrix with entries

$$A_{i,j} = \begin{cases} 1 & \text{if } i = j \\ \frac{1}{(i+j)^2} & \text{otherwise} \end{cases}$$

   For $n = 10, 20, 30, \ldots, 90, 100$, pick the right hand side b so that the solution to $Ax = b$ is the vector $x = [1, 2, \ldots, n]^T$ (do this in your program, before calling your subroutine). Then solve the system of equations for the ten values of n and compute the relative error in the computed solution $\widehat{x}$:

$$e = \frac{\|\widehat{x} - x\|_2}{\|x\|_2}.$$

   where

$$\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}.$$

   Notes: The output should be a table with the values of the relative error e for each value of n. Do not print the solution or the matrix.