2. The linear least-squares approximations on $[-1, 1]$ are:

   (a) $P_1(x) = 3.333333 - 2x$
   (b) $P_1(x) = 0.6000025x$
   (c) $P_1(x) = 0.5493063 - 0.2958375x$
   (d) $P_1(x) = 1.175201 + 1.103639x$
   (e) $P_1(x) = 0.4207355 + 0.4353975x$
   (f) $P_1(x) = 0.6479184 + 0.5281226x$

3. The least squares approximations of degree two are:

   (a) $P_2(x) = 2 + 3x + x^2 \equiv f(x)$
   (b) $P_2(x) = 0.4000163 - 2.400054x + 3.000028x^2$
   (c) $P_2(x) = 1.723551 - 0.9313682x + 0.1588827x^2$
   (d) $P_2(x) = 1.167179 + 0.08204442x + 1.458979x^2$
   (e) $P_2(x) = 0.4880058 + 0.8291830x - 0.7375119x^2$
   (f) $P_2(x) = -0.9089523 + 0.6275723x + 0.2597736x^2$

4. The least squares approximation of degree two on $[-1, 1]$ are:

   (a) $P_2(x) = 3 - 2x + 1.000009x^2$
   (b) $P_2(x) = 0.6000025x$
   (c) $P_2(x) = 0.4963454 - 0.2958375x + 0.1588827x^2$
   (d) $P_2(x) = 0.9962918 + 1.103639x + 0.5367282x^2$
   (e) $P_2(x) = 0.4982798 + 0.4353975x - 0.2326330x^2$
   (f) $P_2(x) = 0.6947898 + 0.5281226x - 0.1406141x^2$

5. The errors E for the least squares approximations in Exercise 3 are:

   (a) 0.3427×10^{-9}
   (b) 0.0457142
   (c) 0.000358354
   (d) 0.0106445
   (e) 0.0000134621
   (f) 0.0000967795

6. The errors for the approximations in Exercise 4 are:

   (a) 0
   (b) 0.0457206
   (c) 0.00035851
   (d) 0.0014082
   (e) 0.00575753
   (f) 0.00011949

7. The Gram-Schmidt process produces the following collections of polynomials:

   (a) $\phi_0(x) = 1, \phi_1(x) = x - 0.5,\quad \phi_2(x) = x^2 - x + \frac{1}{6},$ and $\phi_3(x) = x^3 - 1.5x^2 + 0.6x - 0.05$
   (b) $\phi_0(x) = 1, \phi_1(x) = x - 1,\quad \phi_2(x) = x^2 - 2x + \frac{2}{3},$ and $\phi_3(x) = x^3 - 3x^2 + \frac{12}{5}x - \frac{2}{5}$
   (c) $\phi_0(x) = 1, \phi_1(x) = x - 2,\quad \phi_2(x) = x^2 - 4x + \frac{11}{3},$ and $\phi_3(x) = x^3 - 6x^2 + 11.4x - 6.8$

8. The Gram-Schmidt process produces the following collections of polynomials.

(a) $3.833333\phi_0(x) + 4.000000\phi_1(x)$ (b) $2\phi_0(x) + 3.6\phi_1(x)$

(c) $0.5493061\phi_0(x) - 0.2958369\phi_1(x)$ (d) $3.194528\phi_0(x) + 3\phi_1(x)$

(e) $0.6567600\phi_0(x) + 0.09167105\phi_1(x)$ (f) $1.471878\phi_0(x) + 1.666667\phi_1(x)$

9. The least-squares polynomials of degree two are:

(a) $P_2(x) = 3.833333\phi_0(x) + 4\phi_1(x) + 0.9999998\phi_2(x)$
(b) $P_2(x) = 2\phi_0(x) + 3.6\phi_1(x) + 3\phi_2(x)$
(c) $P_2(x) = 0.5493061\phi_0(x) - 0.2958369\phi_1(x) + 0.1588785\phi_2(x)$
(d) $P_2(x) = 3.194528\phi_0(x) + 3\phi_1(x) + 1.458960\phi_2(x)$
(e) $P_2(x) = 0.6567600\phi_0(x) + 0.09167105\phi_1(x) - 0.73751218\phi_2(x)$
(f) $P_2(x) = 1.471878\phi_0(x) + 1.666667\phi_1(x) + 0.2597705\phi_2(x)$

10. The least-squares polynomials of degree three are:

(a) $P_3(x) = 3.833333\phi_0(x) + 4.000000\phi_1(x) + 0.9999998\phi_2(x)$
(b) $P_3(x) = 2\phi_0(x) + 3.6\phi_1(x) + 3\phi_2(x) + \phi_3(x)$
(c) $P_3(x) = 0.5493061\phi_0(x) - 0.2958369\phi_1(x) + 0.1588785\phi_2(x) - 0.08524470\phi_3(x)$
(d) $P_3(x) = 3.194528\phi_0(x) + 3\phi_1(x) + 1.458960\phi_2(x) + 0.4787959\phi_3(x)$
(e) $P_3(x) = 0.6567600\phi_0(x) + 0.09167105\phi_1(x) - 0.7375118\phi_2(x) - 0.1876952\phi_3(x)$
(f) $P_3(x) = 1.471878\phi_0(x) + 1.666667\phi_1(x) + 0.2597705\phi_2(x) - 0.04559611\phi_3(x)$

11. The Laguerre polynomials are $L_1(x) = x-1$, $L_2(x) = x^2-4x+2$ and $L_3(x) = x^3-9x^2+18x-6$.

12. The least-squares polynomials of degrees one, two, and three are:

(a) $2L_0(x) + 4L_1(x) + L_2(x)$
(b) $\frac{1}{2}L_0(x) - \frac{1}{4}L_1(x) + \frac{1}{16}L_2(x) - \frac{1}{96}L_3(x)$
(c) $6L_0(x) + 18L_1(x) + 9L_2(x) + L_3(x)$
(d) $\frac{1}{3}L_0(x) - \frac{2}{9}L_1(x) + \frac{2}{27}L_2(x) - \frac{4}{243}L_3(x)$

13. Let $\{\phi_0(x), \phi_1(x), \ldots, \phi_n(x)\}$ be a linearly independent set of polynomials in $\prod_n$. For each $i = 0, 1, \ldots, n$, let $\phi_i(x) = \sum_{k=0}^n b_{ki}x^k$. Let $Q(x) = \sum_{k=0}^n a_k x^k \in \prod_n$. We want to find constants $c_0, \ldots, c_n$ so that

$$Q(x) = \sum_{i=0}^n c_i\phi_i(x).$$

This equation becomes

$$\sum_{k=0}^n a_k x^k = \sum_{i=0}^n c_i \left(\sum_{k=0}^n b_{ki}x^k\right)$$