Name: Key

Mathematics 108A: Quiz I
July 1, 2008
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Part I. True-False. Circle the best answer to each of the following questions. Each question is worth 2 points.

1. The set of vectors

$$\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$$

is a (linear) subspace of $\mathbb{R}^3$.

TRUE FALSE

2. The set of vectors

$$\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 - 2x_2 + 3x_3 - 4x_4 = 5\}$$

is a subspace of $\mathbb{R}^4$.

TRUE FALSE

3. The set of vectors $(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5$ such that

$$3x_1 + 2x_2 + 7x_3 + x_4 - 2x_5 = 0, \qquad x_1 + 2x_2 - 5x_3 + x_4 + 4x_5 = 0$$

is a subspace of $\mathbb{R}^4$.

TRUE FALSE

4. The set of vectors

$$\{\mathbf{x} \in \mathbb{C}^3 : \mathbf{x} = s(1,0,0) + t(0, i, \sqrt{2}) \text{ for some } s, t \in \mathbb{C}\ \}$$

is a subspace of $\mathbb{C}^3$.

TRUE FALSE

5. Let $\mathbb{R}^\infty$ be the set of infinite sequences $(x_1, x_2, \ldots, x_i, \ldots)$, where each x_i is a real number. The subset

$$W = \{\mathbf{x} = (x_1, x_2, \ldots, x_i, \ldots) \in \mathbb{R}^\infty : x_2 = 2\},$$

is a subspace of $\mathbb{R}^\infty$.

TRUE FALSE

6. Let $M_{m,n}(\mathbb{R})$ denote the space of $m \times n$ matrices over the complexes, regarded as a vector space over $\mathbb{R}$. Then

$$W = \{A \in M_{2,2}(\mathbb{R}) : \det A \neq 0\},$$

is a subspace of $M_{2,2}(\mathbb{R})$.

TRUE FALSE

7. The set of integers $\mathbb{Z}$ with the usual operations of addition and multiplication is not a field because not all elements have multiplicative inverses within $\mathbb{Z}$.

TRUE FALSE

8. Suppose that V is a vector space over $\mathbb{C}$ with addition

$$f : V \times V \to V, \qquad f(\mathbf{v}, \mathbf{w}) = \mathbf{v} + \mathbf{w}$$

and multiplication

$$g : \mathbb{C} \times V \to V, \qquad g(a, \mathbf{v}) = a\mathbf{v}.$$

Since $\mathbb{R} \subset \mathbb{C}$, we can then define

$$h : \mathbb{R} \times V \to V, \qquad h(a, \mathbf{v}) = g(a, \mathbf{v}).$$

With the operations f and h, V is a vector space over $\mathbb{R}$.

TRUE FALSE

Part II. Give complete answers to each of the following questions.

1. (5 points) Give the definition of a **subspace** U of a vector space V over the field F.

A <u>subspace</u> U of a vector space V is a subset $U \subseteq V$ such that

1. $\vec{0} \in U$.
2. $\vec{x}, \vec{y} \in U \Rightarrow \vec{x} + \vec{y} \in U$.
3. $a \in F$ & $\vec{x} \in U \Rightarrow a\vec{x} \in U$.

b. (10 points) Prove that if U and W are subspaces of V, then so is $U \cap W$.

1. $\vec{0} \in U$ and $\vec{0} \in W$, so $\vec{0} \in U \cap W$.
2. If $\vec{x}, \vec{y} \in U \cap W$, then $\vec{x}, \vec{y} \in U$ and $\vec{x}, \vec{y} \in W$, so $\vec{x} + \vec{y} \in U$ and $\vec{x} + \vec{y} \in W$ and hence $\vec{x} + \vec{y} \in U \cap W$.
3. If $a \in F$ and $\vec{x} \in U \cap W$, then $\vec{x} \in U$ and $\vec{x} \in W$, so $a\vec{x} \in U$ and $a\vec{x} \in W$ and hence $a\vec{x} \in U \cap W$.