Name: Key

Mathematics 108A: Practice Quiz A

June 26, 2008

Professor J. Douglas Moore

I. True-False. Circle the best answer to each of the following questions.

1. The set of vectors
$$\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 \geq 0\}$$
is a subspace of $\mathbb{R}^3$.

TRUE FALSE

2. The set of vectors
$$\{(x_1, x_2, x_3) \in \mathbb{R}^3 : 3x_1 + 2x_2 + 7x_3 = 4\}$$
is a subspace of $\mathbb{R}^3$.

TRUE FALSE

3. The set of vectors
$$\{(x_1, x_2, x_3) \in \mathbb{R}^3 : 3x_1 + 2x_2 + 7x_3 = 0\}$$
is a subspace of $\mathbb{R}^3$.

TRUE FALSE

4. The set of vectors $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$ such that
$$3x_1 + 2x_2 + 7x_3 + x_4 = 0, \qquad x_1 + 2x_2 - 5x_3 + x_4 = 0$$
is a subspace of $\mathbb{R}^4$

TRUE FALSE

5. The set of vectors $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$ such that
$$3x_1 + 2x_2 + 7x_3 + x_4 = 7, \qquad x_1 + 2x_2 - 5x_3 + x_4 = 3$$
is a subspace of $\mathbb{R}^4$.

TRUE FALSE

6. The set of vectors

$$\{\mathrm{x} \in \mathbb{R}^4 : \mathrm{x} = s(1,0,3,0) + t(0,1,1,4) \text{ for some } s,t \in \mathbb{R}\ \}$$

is a subspace of $\mathbb{R}^4$.

TRUE FALSE

7. The set of vectors

$$\{\mathrm{x} \in \mathbb{R}^4 : \mathrm{x} = (1,0,0,0) + s(0,1,0,3) + t(0,1,1,6) \text{ for some } s,t \in \mathbb{R}\ \}$$

is a subspace of $\mathbb{R}^4$.

TRUE FALSE

8. If $V = \{$ functions $f : \mathbb{R} \to \mathbb{R}\ \}$, a vector space over $\mathbb{R}$ with addition and scalar multiplication defined by

$$(f+g)(t) = f(t) + g(t), \quad (af)(t) = a(f(t)), \quad \text{for } f,g \in V \text{ and } a \in \mathbb{R},$$

and

$$W = \{f \in V : f(0) = 3\},$$

then W is a subspace of V.

TRUE FALSE

9. If V is the vector space described in the previous problem and

$$W = \{f \in V : f(3) = 0\},$$

then W is a subspace of V.

TRUE FALSE

10. Suppose that $V = \{$ differentiable functions $f : \mathbb{R} \to \mathbb{R}\ \}$, a vector space over $\mathbb{R}$ with addition and scalar multiplication defined by

$$(f+g)(t) = f(t) + g(t), \quad (af)(t) = a(f(t)), \quad \text{for } f,g \in V \text{ and } a \in \mathbb{R},$$

and

$$W = \{f \in V : f''(t) + 4f(t) = 0\},$$

then W is a subspace of V.

TRUE FALSE

11. Any vector space over $\mathbb{C}$ can also be thought of as a vector space over $\mathbb{R}$.

TRUE FALSE

12. Let $M_{m,n}(\mathbb{C})$ denote the space of $m \times n$ matrices over the complexes, regarded as a vector space over $\mathbb{C}$. If $V = M_{2,2}(\mathbb{C})$ and

$$W = \{A \in M_{2,2}(\mathbb{C}) : A = \bar{A}^T\},$$

where $\bar{A}$ denotes the conjugate of A, then W is a subspace of V

TRUE FALSE

13. If $V = M_{2,2}(\mathbb{C})$ is regarded as a vector space over $\mathbb{R}$ and

$$W = \{A \in M_{2,2}(\mathbb{C}) : A = \bar{A}^T\},$$

then W is a subspace of V.

TRUE FALSE

14. Let

$$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad I = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix},$$

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad K = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

Let $V = M_{2,2}(\mathbb{C})$ regarded as a vector space over $\mathbb{R}$. If

$$\mathbb{H} = \{A \in M_{2,2}(\mathbb{C}) : A = tU + xI + yJ + zK, \text{ where } t, x, y, z \in \mathbb{R}\},$$

then $\mathbb{H}$ is a subspace of V. Hence $\mathbb{H}$ is a vector space over $\mathbb{R}$.

TRUE FALSE

15. If addition is vector addition and multiplication is matrix multiplication, then $\mathbb{H}$ satisfies all of the field axioms except for commutativity of multiplication.

TRUE FALSE

II. Complete answers. Write out complete solutions to each of the following problems.

1. Prove that if U and W are subspaces of V, then so is the intersection $U \cap W$.

2. If U and W are subspaces of V, let

$$U + W = \{u + w : u \in U, w \in W\}$$

Show that $U + W$ is a subspace of V.