Name: Key

Mathematics 108A: Practice Quiz H

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I. True-False. Circle the best answer to each of the following questions.

1. If 3 is a root of the polynomial $p(z)$, then there is a polynomial $q(z)$ such that $p(z) = (z-3)q(z)$.

TRUE FALSE

2. Every nonconstant polynomial with coefficients in $\mathbb{R}$ has a root in $\mathbb{R}$.

TRUE FALSE

3. Every nonconstant polynomial with coefficients in $\mathbb{C}$ has a root in $\mathbb{C}$.

TRUE FALSE

4. Let $\beta = (\mathbf{v}_1, \mathbf{v}_2)$ be the basis for $\mathbb{R}^2$ defined by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map such that $T(\mathbf{v}_1) = 3\mathbf{v}_1$ and $T(\mathbf{v}_2) = 7\mathbf{v}_2$, then

$$\mathcal{M}(T, \beta, \beta) = \mathcal{M}(T, (\mathbf{v}_1, \mathbf{v}_2), (\mathbf{v}_1, \mathbf{v}_2)) = \begin{pmatrix} 3 & 0 \\ 0 & 7 \end{pmatrix}.$$

TRUE FALSE

5 The linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

has no real eigenvalues.

TRUE FALSE

6. The linear map $T : \mathbb{C}^2 \to \mathbb{C}^2$ defined by

$$T(\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

has eigenvalues $\pm i$, where $i = \sqrt{-1}$.

TRUE FALSE

7. Let $V = \{$ differentiable functions $f : \mathbb{R} \to \mathbb{R}$ $\}$, a vector space over $\mathbb{R}$ with addition and scalar multiplication defined by

$$(f+g)(t) = f(t) + g(t), \quad (af)(t) = a(f(t)), \quad \text{for } f, g \in V \text{ and } a \in \mathbb{R},$$

and consider the linear map

$$T : V \to \mathbb{R}, \quad \text{defined by} \quad T(f)(x) = \frac{df}{dx}(x).$$

Then the eigenvectors for eigenvalue λ for this operator T are the solutions to the differential equation

$$\frac{df}{dx}(x) = \lambda f(x).$$

TRUE FALSE

II. Complete answers. Write out complete solutions to each of the following problems.

1. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T_A(\mathbf{x}) = A\mathbf{x}, \quad \text{where} \quad A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}.$$

Find a basis for the eigenspace

$$W_3 = \{\mathbf{x} \in \mathbb{R}^3 : T_A(\mathbf{x}) = 3\mathbf{x}\}.$$

$$T_A(\vec{x}) = 3\vec{x} \iff (A - 3I)\vec{x} = \vec{0} \iff \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \vec{0}$$

$$\iff x_2 = 0 \ \& \ x_3 = 0$$

$$W_3 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : x_2 = x_3 = 0 \right\} = \text{span} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Basis for W_3: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.