Name: Key

Mathematics 108A: Practice Quiz J

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Part I. Multiple Choice. Circle the best answer to each of the following questions.

1. Suppose that V and W are complex vector spaces of dimension 5 and 6 respectively, and that $T : V \to W$ is a linear transformation such that null(T) has dimension 3. Then the dimension of range(T) is

a. 2   b. 3   c. 5   d. 6   e. None of these

2. Suppose that the matrix of a linear transformation $T : V \to V$ with respect to a basis $(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ is

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}.$$

Then the eigenvalues of T are...

a. 2   b. 3   c. 2 and 3   d. 1, 2 and 3   e. None of these

3. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and that

$$T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad T\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

If $(\mathbf{v}_1, \mathbf{v}_2)$ is the basis for $\mathbb{R}^2$ defined by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix},$$

then the matrix of T with respect to this basis is...

a. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ b. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ c. $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

d. $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ e. None of these

Part II. Give complete answers to each of the following questions.

1. Let $T_A : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by the matrix

$$\begin{pmatrix} 1 & 0 & -2 & -1 \\ 3 & 0 & -6 & -3 \\ 2 & 0 & -4 & -2 \end{pmatrix} \in \mathrm{Mat}(3,4,\mathbb{R}).$$

a. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the rows of A.

Row reduced echelon form $\begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

If W = subspace of $\mathbb{R}^4$ spanned by columns of A,

Basis for W is $(1 \ 0 \ -2 \ -1)$

b. Find a basis for the null space of T_A.

$x_1 \quad -2x_3 - x_4 = 0$

$$\begin{aligned} x_1 &= 2x_3 + x_4 \\ x_2 &= x_2 \\ x_3 &= x_3 \\ x_4 &= x_4 \end{aligned}$$

Basis for null(T_A)

$$\vec{x} = x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

c. Find a basis for the range of T_A.

Column reduced echelon form $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{pmatrix}$

Basis for range T_A : $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$.

2. Let $T_A : \mathbb{R}^5 \to \mathbb{R}^3$ be the linear transformation defined by the matrix

$$\begin{pmatrix} 1 & 3 & 0 & 4 & 6 \\ 1 & 3 & 1 & 5 & 8 \\ 2 & 6 & 0 & 8 & 12 \end{pmatrix} \in \mathrm{Mat}(3,5,\mathbb{R}).$$

a. Find a basis for the subspace of $\mathbb{R}^5$ spanned by the rows of A.

Row-reduced echelon form $\begin{pmatrix} 1 & 3 & 0 & 4 & 6 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Basis for span of rows of A: $\left\{ \begin{matrix} (1 & 3 & 0 & 4 & 6) \\ (0 & 0 & 1 & 1 & 2) \end{matrix} \right.$

b. Find a basis for the null space of T_A.

$$\begin{aligned} x_1 &= -3x_2 - 4x_4 - 6x_5 \\ x_2 &= x_2 \\ x_3 &= -x_4 - 2x_5 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}$$

Basis for null(T_A)

$$\vec{x} = x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -4 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -6 \\ 0 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$

c. Find a basis for the range of T_A.

Column reduced echelon form: $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{pmatrix}$

Basis for Range (T_A): $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$