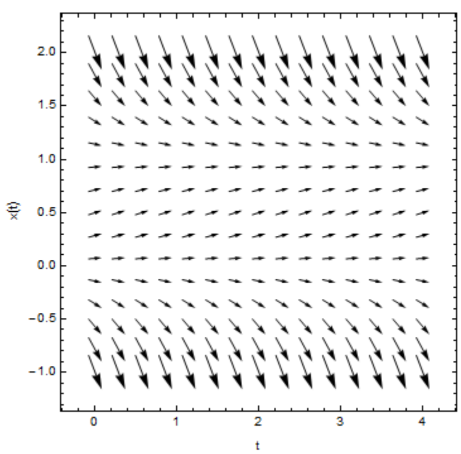
Slope Fields

The Punch Line: Slope fields allow us to visualize the behavior of a differential equation (without solving it).

1: Suppose we have the differential equation

$$x' = x(1 - x),$$

involving some function $x(t)$. It has the *slope field* below:



Does this equation have solutions which

(a) constantly grow in magnitude (absolute value)?

(b) constantly shrink in magnitude?

(c) constantly grow larger (more positive)?

(d) constantly grow smaller (more negative)?

(e) are bounded (magnitude is smaller than some value for all time)?

(f) are unbounded (magnitude grows arbitrarily large)?

(g) are constant? (these are *equilibrium solutions*)

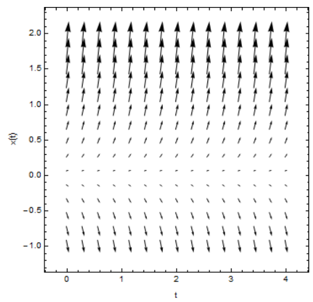
(h) grow arbitrarily large in the positive direction?

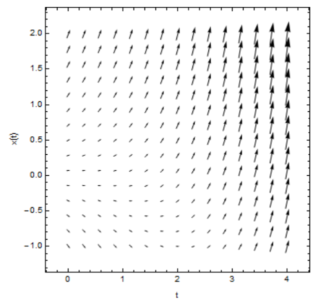
(i) grow arbitrarily large in the negative direction?

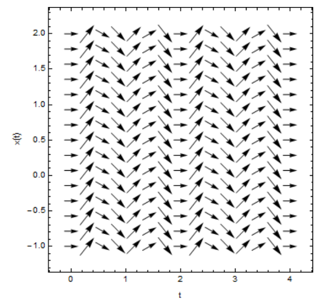
Try to describe or sketch any solutions which have these behaviors. Where in the plot do they occur?

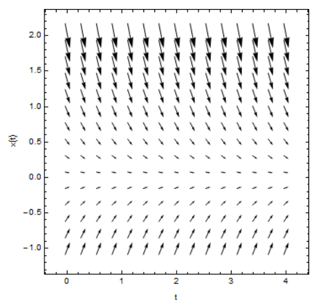
Don't worry if you can't wholly justify your answers, or if you aren't able to tell the behavior from the plot definitively—we don't have the analytical tools to properly answer these questions yet, this is just a chance to think about what kind of information we can get from a slope field!

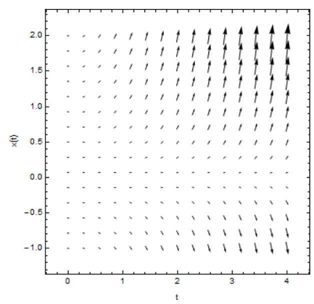
2: Match the slope fields below to the given differential equations:

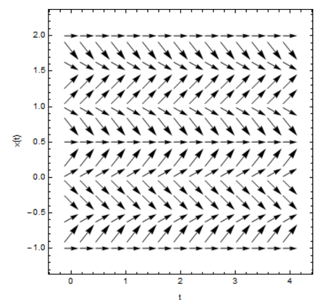

(a)


(c)


(e)


(b)


(d)


(f)

(i) $x' = x + \frac{t^2}{4}$

(iii) $x' = tx$

(v) $x' = \sin(2\pi x)$

(ii) $x' = 4x$

(iv) $x' = -2x$

(vi) $x' = \sin(2\pi t)$