Math 4B
Worksheet 4b

Second Order Equations I

The Punch Line: The solutions to many second-order constant-coefficient equations may be deduced by solving quadratic equations.

Linear Homogeneous Constant-Coefficient Second Order Equations: If we have the differential equation $ay'' + by' + cy = 0$, we can ask ourselves if there are any solutions of the form $y(t) = e^{rt}$. If so, we would have $ar^2e^{rt} + bre^{rt} + ce^{rt} = 0$, which can only happen if $ar^2 + br + c = 0$ ($e^{rt} > 0$ for all t). This quadratic equation is the *characteristic equation*, and its roots tell us what solutions to expect. In general, the solution to an IVP is a linear combination of the two possibilities (if there are two distinct roots), with coefficients chosen to match the initial conditions.

1: Solve the following differential equations with the initial conditions $y(0) = 1$ and $y'(0) = 0$:

(a) $y'' - 9y = 0$

(b) $y'' - 3y' + 2y = 0$

(c) $2y'' + 3y' + y = 0$

(d) $y'' + y' = 0$

Challenge Problems: Sometimes, the solution technique above will fail (e.g., what happens if the characteristic equation has only one root?). Also, we are occasionally interested in *boundary value problems*, where instead of a derivative condition we have a value condition at a different point. The ideas used in dealing with these situations are similar (although we haven't done them yet in class, so don't worry if you can't solve these problems).

Challenges: Solve these differential equations

(a) $y'' - 4y' + 4y = 0$, where $y(0) = 1$, $y'(0) = 0$ (hint: think about $y'' = 0$ first)

(b) $y'' - y = 0$, where $y(0) = 1$ and $y(1) = 0$

(c) $y'' + y = 0$, where $y(0) = y(\pi) = 0$