Spring Systems

The Punch Line: One of the most common DE models is the linear mass-spring system.

Setup: The mass-spring-damper system has the general form $mu'' + \gamma u' + ku = F(t)$, where m, γ, and k are the mass, damping, and spring constants, respectively, and $F(t)$ is an external force. We divide these models into three classes based on the discriminant $\gamma^2 - 4mk$: *underdamped* if $\gamma < \sqrt{4mk}$ (complex roots), *critically damped* if $\gamma = \sqrt{4mk}$ (repeated root), and *overdamped* if $\gamma > \sqrt{4mk}$ (distinct real roots).

1: (Note: in the given units, the gravitational acceleration is 9.8 meters per second squared.)

(a) A mass of 10 kilograms is placed on a scale supported by a spring, pushing it down by 1.4 meters (it is a very large scale). Material imperfections in the spring cause it to resist the motion of the mass with a force of 240 newtons (kilogram meters per second squared) when it has a velocity of 3 meters per second squared.

What is the general solution to this DE? Is it under-, over-, or critically damped?

(b) A particular spring has a spring constant of 100 newtons per meter (kilograms per second squared) and damping constant of 60 newtons per meter per second (kilograms per second). When a particular mass is suspended from this spring, it stretches by 98 centimeters.

What is the general solution to this DE? Is it under-, over-, or critically damped?

(c) A mass of 2 kilograms is suspended from a spring, stretching it by 2.45 meters. You are in charge of choosing a damping constant to make this system critically damped. What damping constant do you choose? What is the general solution to the resulting differential equation?