Math 6B
Quiz 1

Quiz–Green's Theorem

Let $\vec{c}$ be a positively oriented path along the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$. Compute

$$\int_{\vec{c}} e^y\,dx + e^x\,dy.$$

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with algebraic errors.

We first compute $\operatorname{curl}(\vec{F}) = e^x - e^y$. Then we compute via Green's Theorem

$$\begin{aligned}
\int_{\vec{c}} e^y\,dx + e^x\,dy &= \int_0^1 \int_0^{2-2x} e^x - e^y\,dy\,dx \\
&= \int_0^1 2e^x - 2xe^x - e^{2-2x} + 1\,dx \\
&= 2\left[e^x\right]_0^1 - 2\left[xe^x\right]_0^1 + 2\int_0^1 e^x\,dx + \frac{1}{2}\left[e^{2-2x}\right]_0^1 + \left[x\right]_0^1 \\
&= 2e - 2 - 2e + 0 + 2e - 2 + \frac{1}{2} - \frac{1}{2}e^2 + 1 - 0 \\
&= 2e - \frac{5}{2} - \frac{1}{2}e^2.
\end{aligned}$$

Alternately, we can see the integral along the horizontal leg is 1 and along the vertical leg is -2, and the integral along the hypoteneuse is

$$\int_0^1 (e^{2t}, e^{1-t})\cdot(-1,2)\,dt = \int_0^1 2e^{1-t} - e^{2t}\,dt = \left[-2e^{1-t} - \frac{1}{2}e^{2t}\right]_0^1 = -2 - \frac{1}{2}e^2 + 2e + \frac{1}{2},$$

so putting these together gives the final integral

$$\int_{\vec{c}} e^y\,dx + e^x\,dy = 2e - \frac{5}{2} - \frac{1}{2}e^2.$$