Math 6B
Quiz 2

Quiz–Stokes' Theorem

Let $\vec{c}(t)$ be a curve around the irregular pentagon in the $z = 1$ plane with vertices $(0,0,1)$, $(0,1,1)$, $(1,2,1)$, $(2,1,1)$, and $(2,0,1)$ oriented according to the upward unit normal, and $\vec{F}$ be the vector field (x, y, xy). Compute $\int_{\vec{c}} \vec{F} \cdot d\vec{s}$.

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with computational errors.

We first compute $\nabla \times \vec{F} = (x, -y, 0)$. We then use Stokes' Theorem to write

$$\int_{\vec{c}} \vec{F} \cdot d\vec{s} = \iint_S \nabla \times \vec{F} \cdot \hat{n}\, dA,$$

and since $\nabla \times \vec{F}$ is orthogonal to the upward unit normal, the integrand is zero, hence the integral is zero.