

Quiz–Sequences

Suppose that the nonnegative increasing sequence a_n converges to L (that is, $\lim_{n \rightarrow \infty} a_n = L$). Use the definition of limit to show that $\{a_n\}$ is bounded (there exists some number $M > 0$ such that $|a_n| < M$ for all n), or give an example of a convergent nonnegative increasing sequence which is not bounded.

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with computational errors.

Since the sequence is increasing, we know $a_n \leq L$ for all n (if $a_n > L$ for some n , the sequence would have to be greater than L for all n afterwards, a contradiction). Since the sequence is nonnegative, we know $0 \leq a_n$ for all n . Thus, we know that $|a_n| = a_n \leq L$, so the sequence is bounded by $M = L + 1$.