Quiz–Series

Determine if the series

$$\sum_{n=0}^{\infty} 3^{-n}(2^n - 2^{-n})$$

is convergent or divergent. If it is convergent, compute its value.

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with computational errors.

We write the series as

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n - \left(\frac{1}{6}\right)^n,$$

which is the sum of two geometric series, so it has the sum

$$\frac{1}{1 - \frac{2}{3}} - \frac{1}{1 - \frac{1}{6}} = 3 - \frac{6}{5} = \frac{9}{5}.$$