Math 6B
Quiz 6

Quiz–Power Series

The function $f(x) = \sum_{k=1}^{\infty} \frac{1}{k} x^k$ is defined on $|x| < 1$.

(a) Compute a power series representation of $f'(x)$.

(b) What is $f(x)$ as a function?

(c) What is $\int_0^x f(x)\,dx$? Compute this as a power series, then try to interpret it as a function.

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with computational errors.

Differentiating term by term, we get

$$f'(x) = \sum_{k=1}^{\infty} x^{k-1} = \sum_{j=0}^{\infty} x^j.$$

We recognize this as the series for $\frac{1}{1-x}$, which helps us recognize $f(x) = -\ln(1-x)$. Integrating term-by-term, we get

$$\int_0^x f(x)\,dx = \sum_{k=1}^{\infty} \frac{1}{k(k+1)} x^{k+1} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right) x^{k+1} = x \sum_{k=1}^{\infty} \frac{1}{k} x^k - \sum_{j=2}^{\infty} \frac{1}{j} x^j = -x\ln(1-x) + \ln(1-x) - x.$$