Math 6B
Quiz 7

Quiz–Fourier Series

Let f be the function defined by

$$f(x) = \begin{cases} 0, & -1 \le x \le 0, \\ x, & 0 \le x \le 1 \end{cases}.$$

(a) Is f even, odd, both, or neither?

(b) Compute the Fourier series for f.

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with computational errors.

(a) This function is neither even nor odd.

(b) The mean value of f is $a_0 = \frac{1}{4}$ (consider the triangle). From the above, we need both the cosine and the sine series. We compute

$$\begin{aligned} a_k &= \int_0^1 x\cos(k\pi x)\,dx \\ &= \left[\frac{1}{k\pi}x\sin(k\pi x)\right]_0^1 - \int_0^1 \frac{1}{k\pi}\sin(k\pi x)\,dx \\ &= \left[\frac{1}{k^2\pi^2}\cos(k\pi x)\right]_0^1 = \frac{(-1)^k - 1}{k^2\pi^2}, \\ b_k &= \int_0^1 x\sin(k\pi x)\,dx \\ &= \left[\frac{1}{k\pi}x\cos(k\pi x)\right]_0^1 + \int_0^1 \frac{1}{k\pi}\cos(k\pi x)\,dx \\ &= \frac{(-1)^k}{k\pi} + \left[\frac{1}{k^2\pi^2}\sin(k\pi x)\right]_0^1 = \frac{(-1)^k}{k\pi}. \end{aligned}$$

Thus, the Fourier series is

$$\frac{1}{4} + \sum_{k=1}^{\infty}\left(\frac{(-1)^k - 1}{k^2\pi^2}\right)\cos(k\pi x) + \frac{(-1)^k}{k\pi}\sin(k\pi x).$$