Math 6B
Quiz 8

Quiz–Fourier Transform

(a) What is the Fourier transform of the function f defined by

$$f(x) = \begin{cases} 1, & -1 \le x \le 1, \\ 0, & \text{else} \end{cases}$$

(b) What is the Fourier transform of the function g defined by

$$g(x) = \begin{cases} x, & -1 \le x \le 1, \\ 0 & \text{else} \end{cases}$$

[Hint: you can use your previous answer and an identity to avoid a second integration.]

Show all work and clearly mark your final answer. No calculators/notes allowed. Partial credit will be given for correctly explaining any steps you're unable to carry out, as well as demonstrating correct methods with computational errors.

(a) We compute

$$F[f](\omega) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-i\omega x}\, dx = \frac{1}{\sqrt{2\pi}} \left[\frac{1}{-i\omega} e^{-i\omega x} \right]_{-1}^{1} \frac{2}{\omega} \left(\frac{e^{i\omega} - e^{-i\omega}}{2i} \right) = \frac{2}{\sqrt{2\pi}} \frac{\sin \omega}{\omega}.$$

(b) We know that $g(x) = xf(x)$, so

$$F[g](\omega) = F[xf](\omega) = iF[-ixf](\omega) = iF[f]'(\omega) = \frac{2i}{\sqrt{2\pi}} \left(\frac{\cos \omega}{\omega} - \frac{\sin \omega}{\omega^2} \right).$$

We could also have computed directly, of course, which is not much harder. We could *not* have used the derivative of g, which fails to exist at $x = \pm 1$.