Series I

The Punch Line: Series are analyzed in terms of their partial sums.

A *series* is the sum of a sequence, which is really a sequence of partial sums. If the partial sums converge, we say the series converges; likewise if it diverges. Important series are *geometric series* of the form $\sum ar^n$, and *telescoping series* of the form $\sum a_{n+1} - a_n$.

Computational Compute the limits of the given series, or prove that they diverge.

(a) $\sum_{n=3}^{\infty} \left(\frac{e}{\pi}\right)^n$

(b) $\sum_{n=5}^{\infty} ne^n$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2+1} - \frac{1}{n^2+2n+2}$

(d) $\sum_{n=1}^{\infty} \frac{2^{3-2n}+2^{2-3n}}{5^{1-n}}$

(e) $\sum_{n=1}^{\infty} 2^{-n} \sin(n\pi/2)$

(f) $\sum_{n=7}^{\infty} \frac{2}{n^2-1}$

Theoretical

(a) Suppose $\sum_{n=0}^{\infty} a_n$ converges. Does the series $\sum_{n=N}^{\infty} a_n$ always converge for fixed $N > 0$? What can we say about its limit when it does?

(b) Suppose $\sum_{n=0}^{\infty} a_n$ converges. Does the sequence $R_N = \sum_{n=N}^{\infty} a_n$ always converge? What can we say about its limit when it does?

(c) Suppose a_n is a sequence of positive real numbers. Can $\sum_{N=0}^{\infty} \sum_{n=0}^{N} a_n$ converge?