Series II

The Punch Line: We can apply various tests to attempt to determine convergence. Important ones are the ratio test (examining $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$) and the root test (examining $\lim_{n\to\infty} \sqrt[n]{a_n}$) for series with positive terms, and the integral test (examining $\int_N^\infty f\,dx$) for series with positive decreasing terms.

Computational Compute the limits of the given series, or prove that they diverge.

(a) $\sum_{n=0}^{\infty} \frac{(2n)!}{(2n)^n}$

(b) $\sum_{n=1}^{\infty} n^{-2n}$

(c) $\sum_{n=1}^{\infty} \frac{1+2n}{(n+1)^\pi}$

(d) $\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2 \ln n}$

Challenging

(a) $\sum_{n=1}^{\infty} \frac{\sum_{k=1}^{n-1} k^{-2}}{\sum_{k=n}^{\infty} k^{-2}}$

(b) $\sum_{n=2}^{\infty} \sum_{m=n}^{\infty} \left(\frac{1}{n}\right)^{m}$