PDEs

The Punch Line: For *separable* partial differential equations, we can write solutions as sums of product solutions (that is, for two dimensional solutions, writing $u(x,t) = G(x)T(t)$). If the differential equation is suitable, this allows us to write ordinary differential equations for the different factors in terms of various parameters, which we can solve to find an overall solution. Various integrals allow us to match the initial conditions.

Computational Solve the following initial value-boundary problems:

(a) On the region $0 < x < 1$, the wave equation

$$u_{tt} = 25u_{xx}$$

with zero boundary conditions, initial condition $u(x,0) = x(1-x)$, and initial derivative $u_t(x,0) = \sin(\pi x)$.

(b) On the region $0 < x < \pi$, the heat equation

$$u_t = 4u_{xx}$$

with zero boundary conditions, and initial condition $u(x,0) = \sin(x) + \sin(3x)$.

Theoretical

(a) Use the Fourier transform (in x) to solve the wave equation

$$u_{tt} = u_{xx}$$

on $\mathbb{R}$ with the initial conditions $u(x,0) = e^{-x^2}$, $u_t(x,0) = 0$. You may use that the Fourier transform of e^{-x^2} is $\frac{1}{\sqrt{2}} e^{-\omega^2/4}$, and leave your answer as an integral.

(b) Use the substitution $v = xu$ to solve the differential equation

$$xu_{tt} = xu_{xx} + 2u_x$$

on $0 < x < \pi$ with zero boundary and initial conditions $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$.