Logical Connectives

Mathematical proofs involve statements (things which are either true or false) which are combined using logical connectives (such as "and," "or," or "not") to form more complicated statements. An argument involves a collection of *premises* (statements which we assume are true), which lead to a conclusion through application of rules of logic. We call a conclusion *valid* if it is true whenever all of the premises are true. As shorthand for representing arguments, we often use letters to stand for simple statements (such as P for "n is a prime number" or G for "$x > 0$"), and symbols for logical connectives ($\wedge$ for "and," $\vee$ for "(inclusive) or," and $\neg$ for "not"). A formula in these symbols is *well-formed* if it can be interpreted as a statement (regardless of whether the statement is true).

1: Analyze the logical form of the following statements as a well-formed formula. Choose letters for simple statements and write down what each represents. Each simple statement should be a single true/false condition.

(a) An integer is either prime or composite.

(b) Seven is both negative and positive.

(c) $|x| \leq 1$ (use statements about x, such as $x < 1$).

(d) I did the reading, or I did not do the reading and the quiz was hard.

(e) Either $f(x) = 0$, or $f(x) \neq 0$ and $\frac{1}{f(x)}$ exists.

2: Which of the following expressions are well-formed formulas? If they are, what is a statement with that logical form (it need not be mathematical)?

(a) $A \wedge (\neg B \vee C)$

(b) $A \neg \wedge B$

(c) $A \wedge \neg B$

(d) $A \vee \neg A$

(e) $\neg\neg A \wedge \neg A$

(f) $\vee A$