Math 8
Worksheet 2

Equivalent Formulas

One of the simplest types of proof is to show that two statements are logically equivalent. In simple cases, this can be done explicitly, for example by constructing the relevant truth tables. Often, it is done by a chain of equivalences: working through intermediate statements which are all equivalent to each other to reach one of the statements from the other. It can sometimes be useful to write out expressions which explicitly encode all necessary conditions, and then reduce these to simpler equivalent expressions which are easier to work with.

1: Find simpler formulas equivalent to these formulas (and prove they are equivalent).
[The first three of these are taken from problem 11 on page 25 of Velleman]

(a) $\neg(\neg P \wedge \neg Q)$

(b) $(P \wedge Q) \vee (P \wedge \neg Q)$

(c) $\neg(P \wedge \neg Q) \vee (\neg P \wedge Q)$

(d) $(\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

(e) $(P \vee R) \wedge (\neg Q \vee R) \wedge (\neg P \vee R \vee Q)$

(f) $\neg\big((\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)\big) \wedge \big((P \vee R) \wedge (\neg Q \vee R) \wedge (\neg P \vee R \vee Q)\big)$

2: Find formulas Φ and Ψ involving A, B, and C with the following truth table:

A	B	C	Φ	Ψ
T	T	T	T	F
T	T	F	F	T
T	F	T	F	T
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F